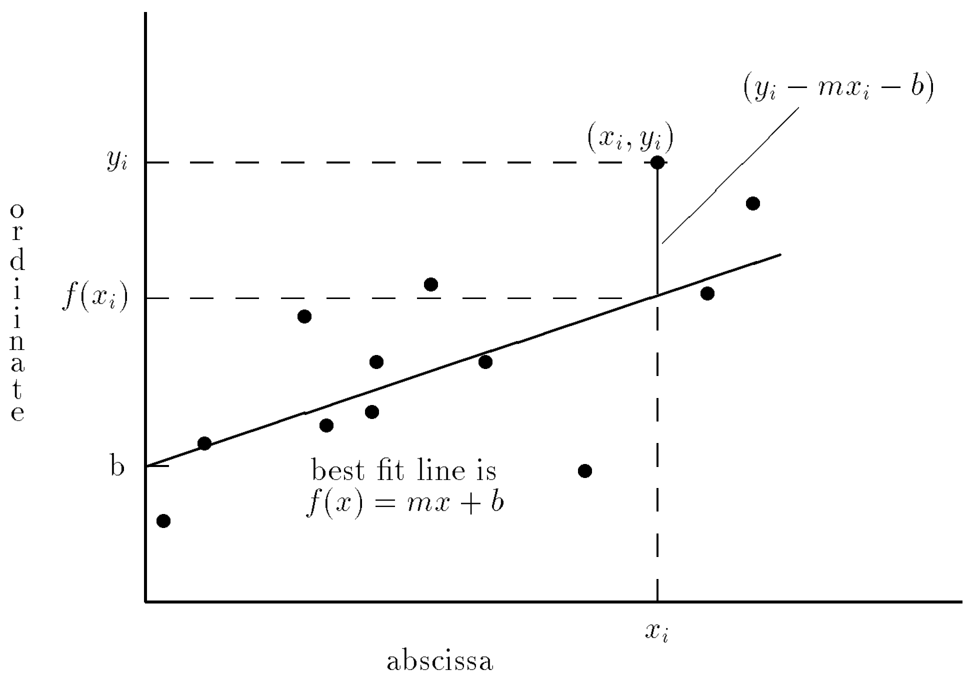
**PHYS 2215** **Method of Least Squares: Linear Case**

Taken from Calculus, eighth edition, by Varberg, Purcell, Rigdon, page 675.

Given $n$ points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ in the $xy-$plane, we wish to find the line $y = mx + b$ such that the sum of the squares of the vertical distances from the points to the line is a minimum; that is, we wish to minimize

$$E(m,b) = \sum_{i=1}^{n} (y_i - mx_i - b)^2. \tag{1}$$

Remember that the $x_i$'s and the $y_i$'s are fixed; they are not unknowns, they are the data values.




Taking derivatives and setting them equal to zero is how we minimize things. Setting $\partial E/\partial m$ and $\partial E/\partial b$ equal to zero leads to the system of two simultaneous equations for $m$ and $b$—the two unknowns in $f(x) = mx + b$. This system is (the student should verify this):

$$m \sum x_i^2 + b \sum x_i = \sum y_i x_i \tag{2}$$

$$m \sum x_i + nb = \sum y_i. \tag{3}$$

The second partials test will show that $f$ is minimized for this choice of $m$ and $b$.

From Calculus, third edition, by Staruss, Bradley, and Smith, page 757.

$$m = \frac{n\sum x_i y_i - \left(\sum x_i\right)\left(\sum y_i\right)}{n\sum x_i^2 - \left(\sum x_i\right)^2} \tag{4}$$

and

$$b = \frac{\sum x_i^2 \sum y_i - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{n\sum x_i^2 - \left(\sum x_i\right)} \tag{5}$$

It can be shown that these values of $m$ and $b$ yield an absolute minimum for $f(m, b)$.