10.2 Exponential Functions

By: Cindy Alder

Objectives:

- Define an exponential function.
- Graph an exponential function.
- Solve exponential equations of the form $a^x = a^k$ for x.
- Use exponential functions in applications involving growth and decay.

Exponential Functions

Exponential Function

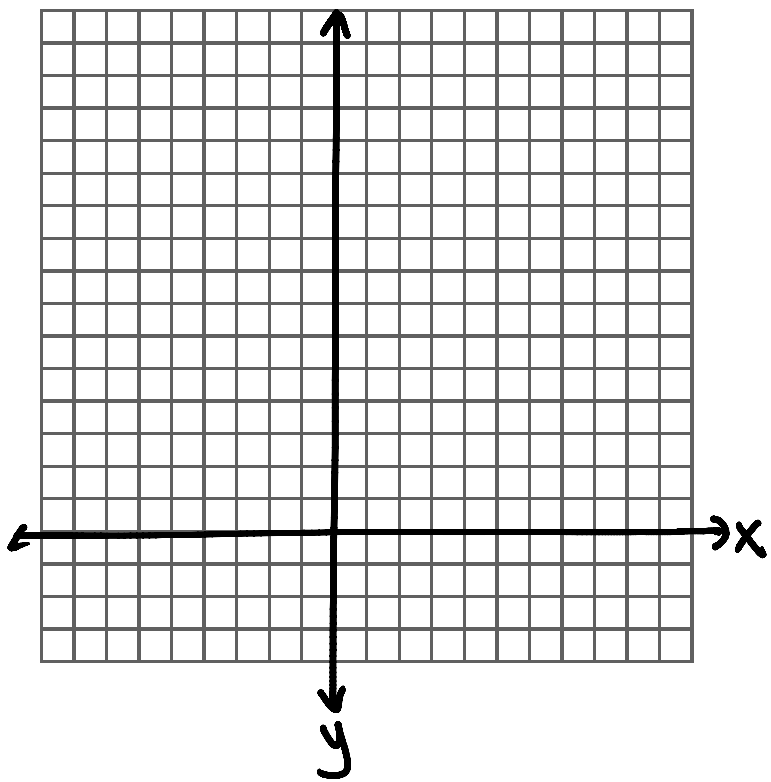
For $a > 0$, $a \neq 1$, and all real numbers x,

defines the exponential function with base $\boldsymbol{a}$.

Graph an Exponential Function $(a > 1)$

$f(x) = 2^x$

$g(x) = 10^x$

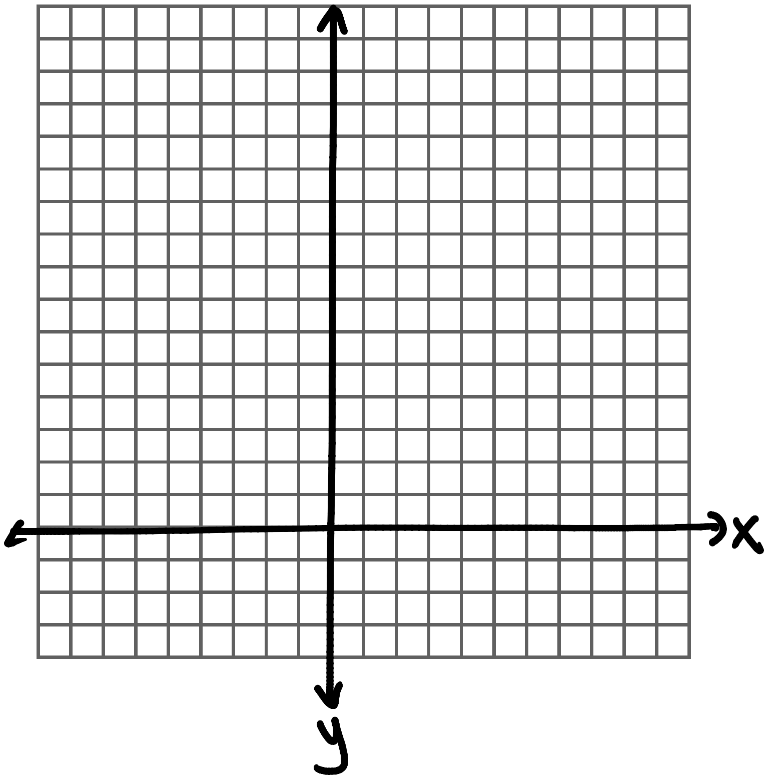


Graph an Exponential Function

$(0 < a < 1)$

$$f(x) = \left(\frac{1}{2}\right)^x$$

$$g(x) = \left(\frac{1}{4}\right)^x$$



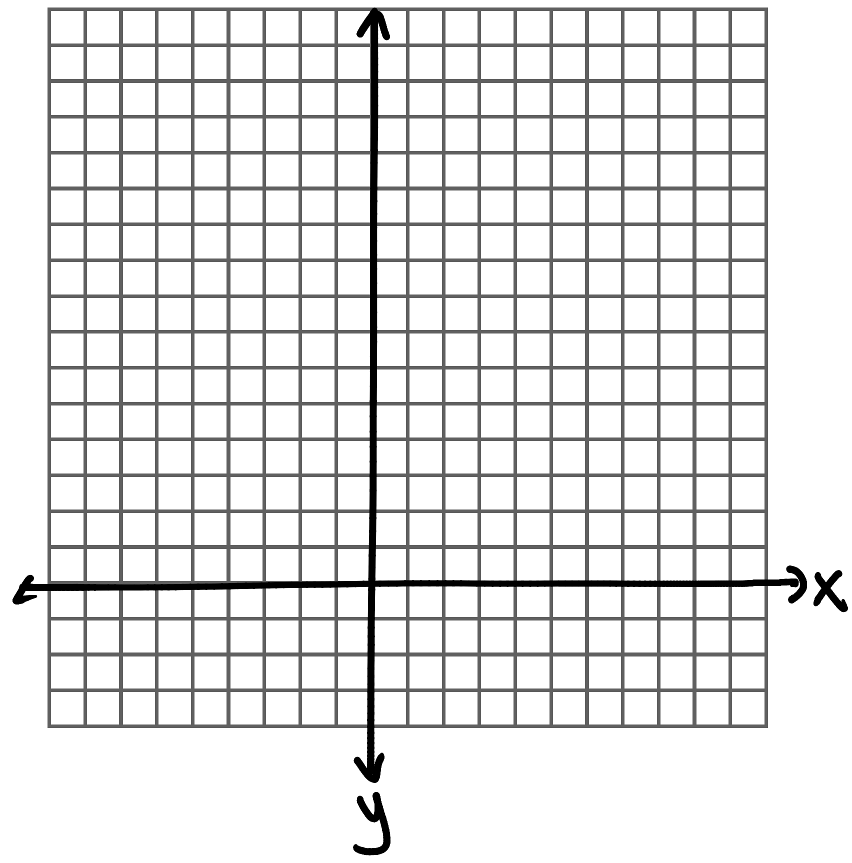
Characteristics of the Graph of $f(x) = a^x$

Exponential Function

- The graph contains the point ______________.
- The function is ____________________.
  - When $a > 1$, the graph will ____________ from left to right.
  - When $0 < a < 1$, the graph will _________ from left to right.
  - In both cases, the graph goes from the ________________________ to the __________.
- The graph will approach the ______________, but never touch it.
  (Such a line is called an ________________)
- The domain is _______________, and the range is ____________.

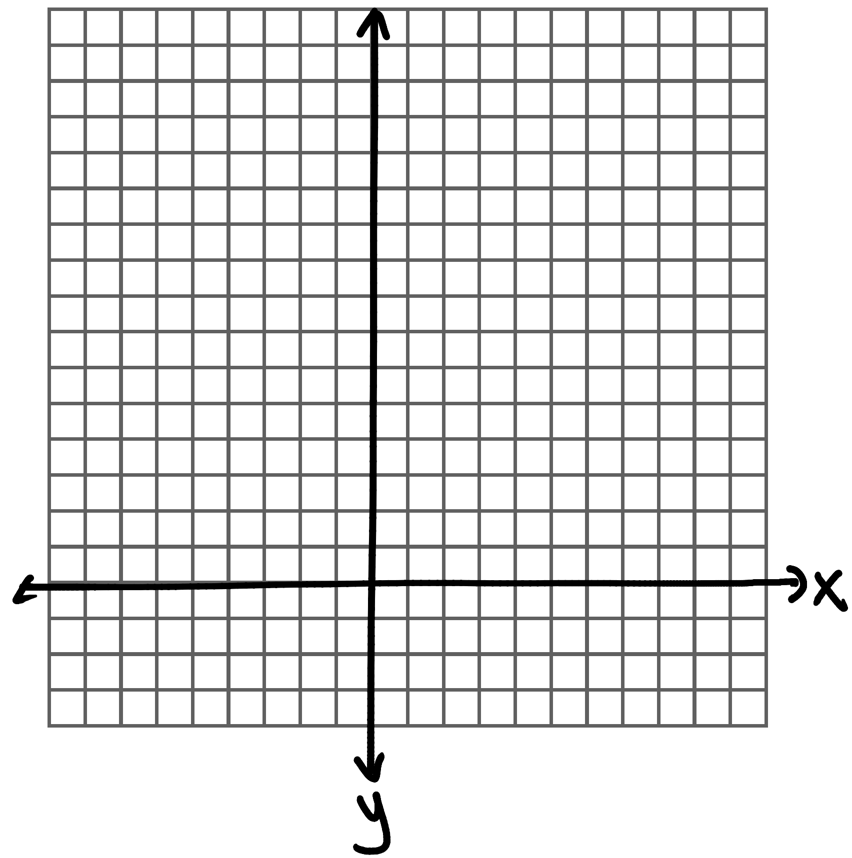
Graphing a More Complicated Exponential Function

$$f(x) = 3^{2x-4}$$



Graphing a More Complicated Exponential Function

$f(x) = 2^{4x-3}$



Solving Exponential Equations

Property for Solving an Exponential Equation

For $a > 0$, $a \neq 1$,

- **Each side must have the same base.**
- **Simplify exponents** if necessary, using the rules of exponents.
- **Set exponents equal** using the property given above.
- **Solve** the equation obtained in previous step.

Solving an Exponential Equation

- **Solve the following equations.**

$$9^x = 27$$

$$25^{x-2} = 125^x$$

Solving an Exponential Equation

- **Solve the following equations.**

$$4^x = \frac{1}{32}$$

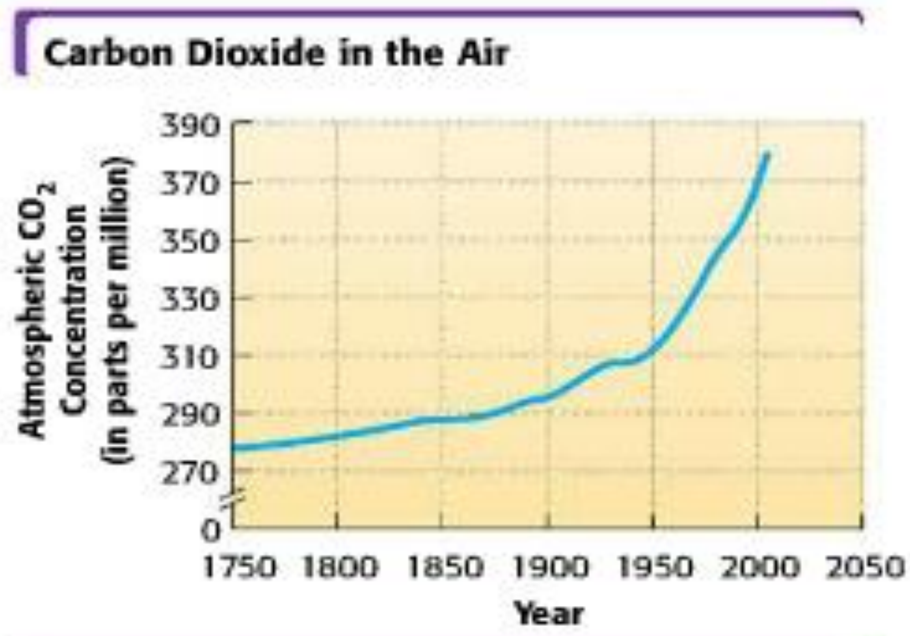
$$\left(\frac{3}{4}\right)^x = \frac{16}{9}$$

Solving an Application Involving Exponential Growth

The graph in FIGURE 8 shows the concentration of carbon dioxide (in parts per million) in the air. This concentration is increasing exponentially. The data are approximated by the function defined by

$$f(x) = 266(1.001)^x$$

where x is the number of years since 1750.



Source: Sacramento Bee; National Oceanic and Atmospheric Administration.

FIGURE 8

Solving an Application Involving Exponential Growth

Use this function and a calculator to approximate the concentration of carbon dioxide in parts per million, to the nearest unit, for the year 2012.

$$f(x) = 266(1.001)^x$$

Applying an Exponential Decay Function

The atmospheric pressure (in millibars) at a given altitude x, in meters, can be approximated by the function defined by

$$\boldsymbol{f(x) = 1038(1.000134)^{-x}}$$

for values between 0 and 10,000.

Because the base is greater than 1 and the coefficient of x in the exponent is negative, function values decrease as x increases. This means that as altitude increases, atmospheric pressure decreases.

Applying an Exponential Decay Function

According to this function, what is the pressure at ground level? Approximate the pressure at 5000 m. Round to the nearest unit.

$$\boldsymbol{f(x) = 1038(1.000134)^{-x}}$$