10.3 Logarithmic Functions
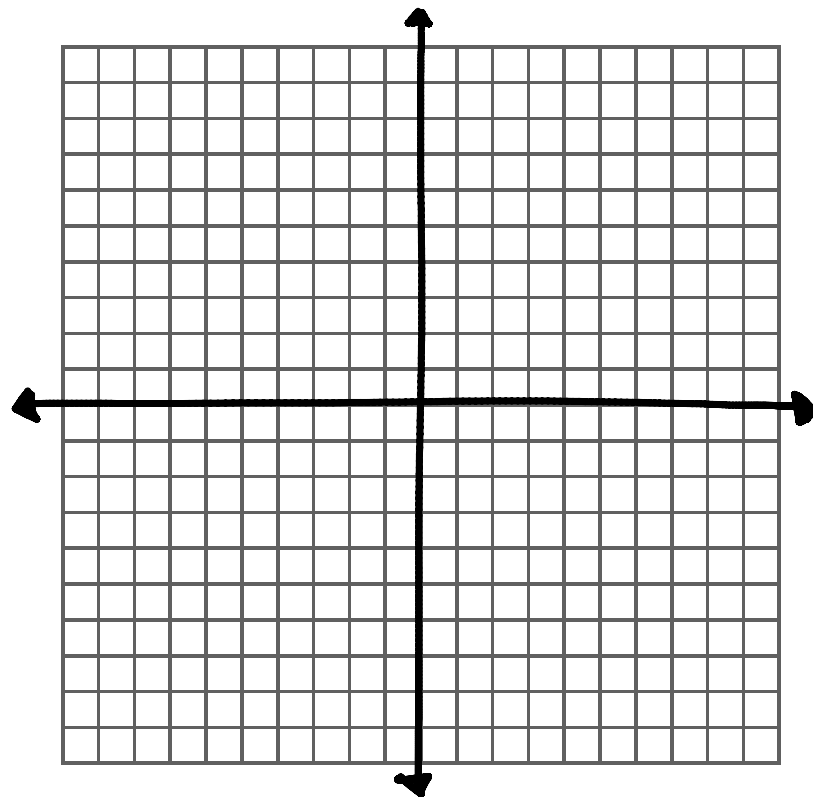
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Objectives:

- Define a logarithm.
- Convert between exponential and logarithmic forms.
- Solve logarithmic equations of the form $\log_a b = k$ for $a, b,$ or k.
- Define and graph logarithmic functions.
- Use logarithmic functions in applications involving growth or decay.

Review

- Graph $y = 2^x$

- Find and graph it's inverse.

Logarithm

For all positive numbers a, with $a \neq 1$, and all positive numbers x

means the same as

Meaning of $\log_a x$

A logarithm is an _____.

The expression $\log_a x$ represents the exponent to which the base a must be raised to obtain x.

Converting Between Exponential and Logarithmic Forms

- Fill in the blanks with the equivalent forms.

Exponential Form	Logarithmic Form
$3^2 = 9$	
$\left(\dfrac{1}{5}\right)^{-2} = 25$	
	$\log_{10} 100{,}000 = 5$
	$\log_4 \dfrac{1}{64} = -3$

- Fill in the blanks with the equivalent forms.

Exponential Form	Logarithmic Form
$2^5 = 32$	
$100^{\frac{1}{2}} = 10$	
	$\log_8 4 = \frac{2}{3}$
	$\log_6 \frac{1}{1296} = -4$
$\sqrt[3]{64} = 4$	

Solving Logarithmic Equations

Solve each equation.

$$\log_4 x = -2 \qquad\qquad \log_{\frac{1}{2}}(3x + 1) = 2$$

Solving Logarithmic Equations

Solve each equation.

$$\log_x 3 = 2$$

$$\log_{49} \sqrt[3]{7} = x$$

Properties of Logarithms

For any positive real number b, with $b \neq 1$, the following are true.

and

• Evaluate each logarithm.

$$\log_7 7 \qquad\qquad\qquad \log_{\sqrt{2}} \sqrt{2}$$

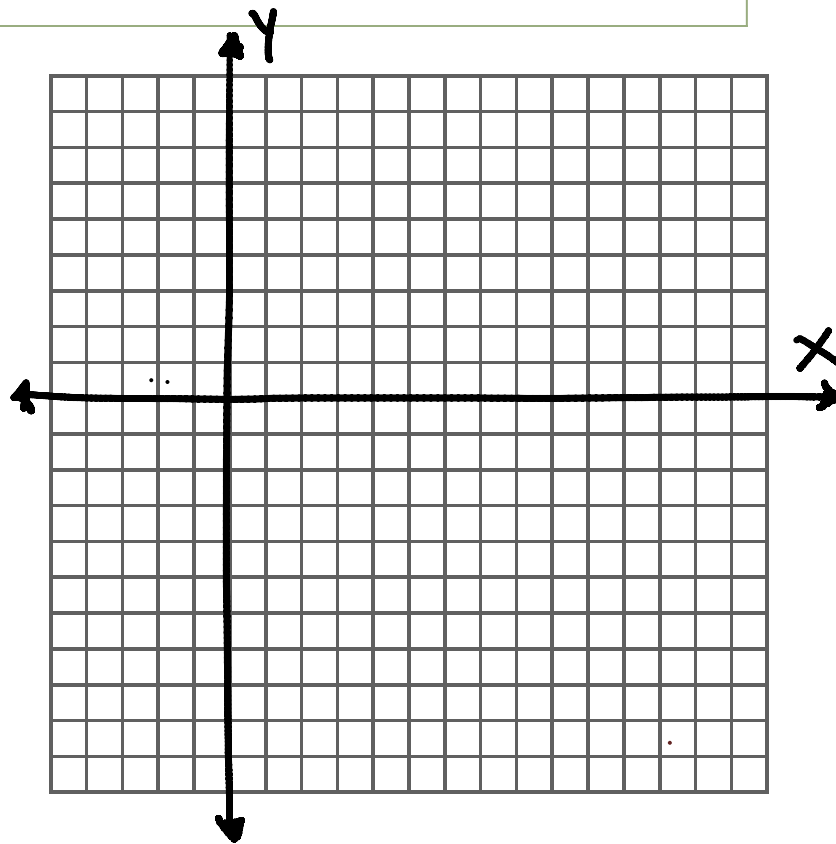$$\log_9 1 \qquad\qquad\qquad \log_{0.2} 1$$

Logarithmic Function

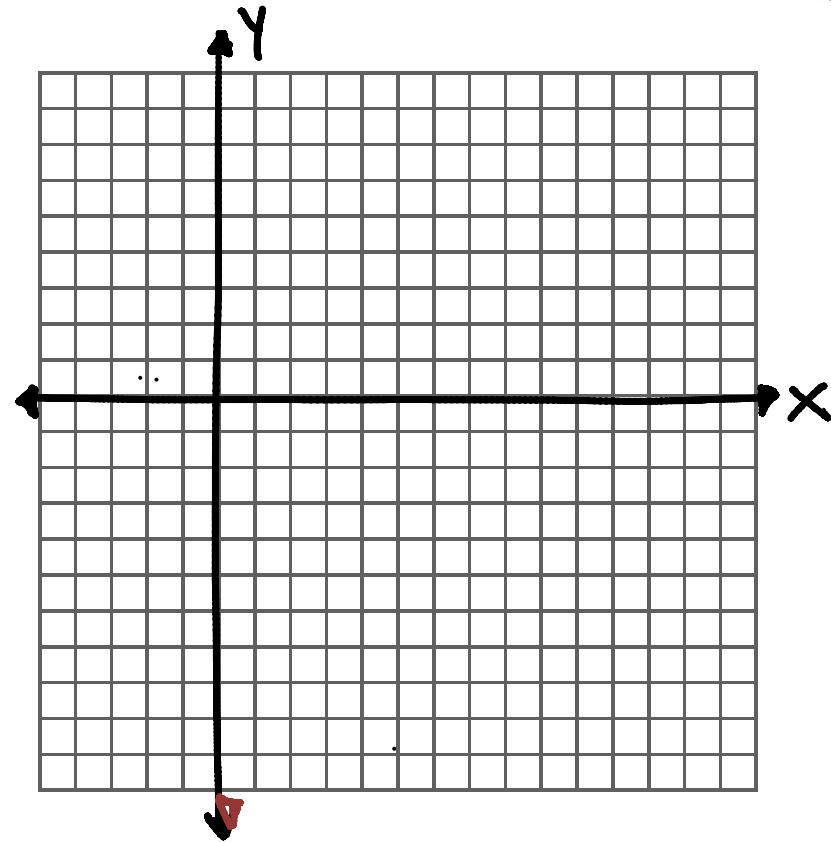If a and x are positive numbers, with $a \neq 1$, then

defines the **logarithmic function with base a**.

- Graph $g(x) = \log_3 x$

Graphing a Logarithmic Function

- Graph $f(x) = \log_{\frac{1}{2}} x$

Characteristics of the Graph of $g(x) = \log_a x$

Logarithmic Function

- The graph contains the point _____.

- The function is _____.
  - When $a > 1$, the graph will _____ from left to right, from the _____.
  - When $0 < a < 1$, the graph will _____ from left to right, from the _____.

- The graph will approach the _____, but never touch it. (The y-axis is an _____.)

- The domain is _____, and the range is _____.

Solving an Application of a Logarithmic Function

- Suppose the gross national product (GNP) of a small country (in millions of dollars) is approximated by

$$G(t) = 15.0 + 2.00 \log_{10} t$$

Where t is the time in years since 2003.

- Approximate to the nearest tenth the GNP for $t = 1$ and $t = 10$.