10.4 Properties of Logarithms

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Objectives:

- Use the product rule for logarithms.
- Use the quotient rule for logarithms.
- Use the power rule for logarithms.
- Use properties to write alternative forms of logarithmic expressions.

Product Rule for Logarithms

If $x, y,$ and b are positive real numbers, where $b \neq 1$, then the following is true.

That is, the logarithm of a product is the sum of the logarithms of the factors.

Using the Product Rule

Use the product rule to rewrite each logarithm. Assume $x > 0$.

$\log_4(3 \cdot 7)$

$\log_8 10 + \log_8 3$

$\log_8 8x$

$\log_5 x^2$

Use the product rule to rewrite each logarithm. Assume $x > 0$.

$\log_5(6 \cdot 9)$

$\log_7 8 + \log_7 12$

$\log_3 3x$

$\log_4 x^3$

Quotient Rule for Logarithms

If $x, y,$ and b are positive real numbers, where $b \neq 1$, then the following is true.

That is, the logarithm of a quotient is the difference between the logarithm of the numerator and the logarithm of the denominator.

- Use the quotient rule to rewrite each logarithm. Assume $\boldsymbol{x > 0}$.

$$\log_7 \frac{9}{4}$$

$$\log_3 p - \log_3 q$$

Using the Quotient Rule

Use the quotient rule to rewrite each logarithm. Assume $x > 0$.

$$\log_4 \frac{3}{16}$$

$$\log_3 \frac{5}{8}$$

Use the quotient rule to rewrite each logarithm. Assume $x > 0$.

$$\log_5 6 - \log_5 x$$

$$\log_3 \frac{27}{5}$$

Rewrite the expression using the properties of logarithms.

$\log_5 2^3$

$\log_2 7^4$

Power Rule for Logarithms

If x, y, and b are positive real numbers, where $b \neq 1$, and if r is any real number, then the following is true.

That is, the logarithm of a number to a power equals the exponent times the logarithm of the number.

Using the Power Rule

Use the power rule to rewrite each logarithm.
Assume $a > 0$, $b > 0$, $x > 0$ and $a \neq 1$

$\log_3 5^2$

$\log_a x^4$

$\log_b \sqrt{8}$

$\log_2 \sqrt[7]{x^4}$

Special Properties

If $b > 0$ and $b \neq 1$, then the following are true.

Using the Special Properties

Find each value.

$\log_{10} 10^5$

$\log_2 8$

$5^{\log_5 9}$

$4^{\log_4 \pi}$

Properties of Logarithms

If x, y, and b are positive real numbers, where $b \neq 0$, and r is any real number, then

NAME OF PROPERTY	PROPERTY	EXAMPLE
Product Rule	$\log_b xy = \log_b x + \log_b y$	$\log_5 (6 \cdot 9) = \log_5 6 + \log_5 9$
Quotient Rule	$\log_b \frac{x}{y} = \log_b x - \log_b y$	$\log_4 \frac{7}{9} = \log_4 7 - \log_4 9$
Power Rule	$\log_b x^r = r \log_b x$	$\log_5 4^2 = 2 \log_5 4$
Special Properties	$b^{\log_b x} = x$ *and* $\log_b b^x = x$	$4^{\log_4 10} = 10$ *and* $\log_5 5^4 = 4$

Writing Logarithms in Alternative Forms

Use the properties of logarithms to rewrite each expression of possible. Assume that all variables represent positive real numbers.

$\log_6 36m^5$

$\log_2 \sqrt{9z}$

Use the properties of logarithms to rewrite each expression if possible. Assume that all variables represent positive real numbers.

$$\log_5 \frac{a^2}{bc}$$

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Use the properties of logarithms to rewrite each expression if possible. Assume that all variables represent positive real numbers.

$$\log_b \frac{8r^2}{m-1}$$

Writing Logarithms in Alternative Forms

Use the properties of logarithms to rewrite each expression as a single logarithm if possible. Assume that all variables represent positive real numbers.

$$4\log_b m - \log_b n$$

Use the properties of logarithms to rewrite each expression as a single logarithm if possible. Assume that all variables represent positive real numbers.

$$2\log_b x + 3\log_b y$$

Use the properties of logarithms to rewrite each expression as a single logarithm if possible. Assume that all variables represent positive real numbers.

$$\log_b(x+1) + \log_b(2x+1) - \frac{2}{3}\log_b x$$

Using the Properties of Logarithms with Numerical Values

Given that $\log_2 5 = 2.3219$ and $\log_2 3 = 1.5850$, evaluate the following.

$\log_2 15$

$\log_2 27$

Given that $\log_2 \mathbf{5} = \mathbf{2.3219}$ and $\log_2 \mathbf{3} = \mathbf{1.5850}$, evaluate the following.

$$\log_2 0.6$$

Deciding Whether Statements About Logarithms Are True or False

Decide whether each statement is *true* or *false.*

$$\log_2 8 - \log_2 4 = \log_2 4$$

Decide whether each statement is *true* or *false*.

$$\log_4(\log_2 16) = \frac{\log_6 6}{\log_6 36}$$

Decide whether each statement is *true* or *false*.

$$\log_3(\log_2 8) = \frac{\log_7 49}{\log_8 64}$$