10.5 Common and Natural Logarithms

By: Cindy Alder

Objectives:

- Evaluate common logarithms using a calculator.
- Use common logarithms in applications.
- Evaluate natural logarithms using a calculator.
- Use natural logarithms in applications.
- Use the change-of-base rule.

Common Logarithms

- Logarithms are important in many applications of mathematics to everyday problems, particularly in biology, engineering, economics, and social science.
- In this section we find numerical approximations for logarithms.
- Traditionally, base 10 logarithms were used most often because our number system is base 10.
- Logarithms to base 10 are called _____ _____.

Evaluating Common Logarithms

Using a calculator, evaluate each logarithm to four decimal places.

$\log 41{,}600$

$\log 43.5$

$\log 0.422$

$\log 0.0615$

Using pH in Application

In chemistry, pH is a measure of the acidity or alkalinity of a solution. Pure water, for example, has pH 7. In general, acids have pH numbers less than 7, and alkaline solutions have pH values greater than 7, as shown in FIGURE 14.

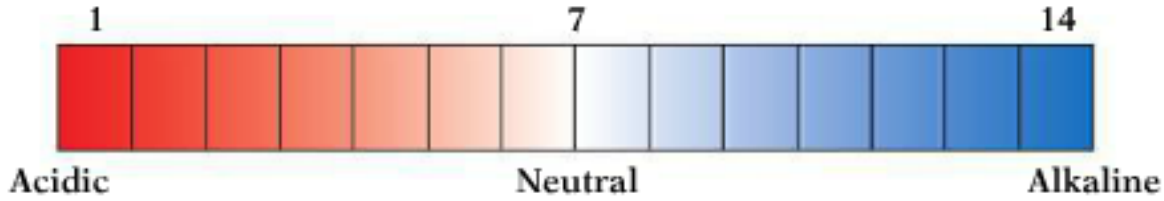

FIGURE 14 pH Scale

The **pH** of a solution is defined as

Using pH in Application

Wetlands are classified as *bogs, fens, marshes,* and *swamps,* on the basis of pH values. A pH value between 6.0 and 7.5, such as that of Summerby Swamp in Michigan's Hiawantha National Forest, indicates that the wetland is a "rich fen." When the pH is between 3.0 and 6.0, the wetland is a "poor fen," and if the pH falls to 3.0 or less, it is a "bog." (*Source:* Mohlenbrock, R., "Summerby Swamp, Michigan." *Natural History.)*

Using pH in Application

Find the pH of water with a hydronium ion concentration of 1.2×10^{-3}. If it is taken from a wetland, is the wetland a rich fen, a poor fen, or a bog?

$$pH = -\log\left[H_3O^+\right]$$

Finding Hydronium Ion Concentration

Find the hydronium ion concentration of a solution with pH 4.6.

$$pH = -\log\left[H_3O^+ \right]$$

Measuring the Loudness of Sound

The loudness of sound is measured in a unit called a **decibel,** abbreviated **dB**.

To measure with this unit, we first assign an intensity of I_0 to a very faint sound, called the **threshold sound.**

If a particular sound has intensity I, then the decibel level of this louder sound is

Measuring the Loudness of Sound

Decibel Level	Example
60	Normal conversation
90	Rush hour traffic, lawn mower
100	Garbage truck, chain saw, pneumatic drill
120	Rock concert, thunderclap
140	Gunshot blast, jet engine
180	Rock launching pad

Source: Deafness Research Foundation

Any sounds over 85 dB exceeds what hearing experts consider safe. Permanent hearing damage can be suffered at levels above 150 dB.

Measuring the Loudness of Sound

Find the decibel level to the nearest whole number of a whisper with intensity I of $115I_0$.

$$D = 10\log\left(\frac{I}{I_0}\right)$$

Natural Logarithms

- The most important logarithms used in applications are _____, which have as a base the number e.

- The number e, like π, is called a **universal constant.** Since it is an irrational number, its decimal expansion never terminates and never repeats.

- The first few digits of the decimal value of e are

Natural Logarithms

- Logarithms to base e are called natural logarithms because they occur in natural situations that involve growth or decay.

- The base e logarithm is written

- This is read as

Evaluating Natural Logarithms

Using a calculator, evaluate each logarithm to four decimal places.

$\ln 0.01$

$\ln 27$

$\ln 529$

$\ln 0.5841$

Applying a Natural Logarithmic Function

The altitude in meters that corresponds to an atmospheric pressure of *x* millibars is given by the logarithmic function defined by

$$f(x) = 51{,}600 - 7457 \ln x$$

Approximate the altitude at 700 millibars of pressure.

Change-of-Base Rule

If $a > 0,\ a \neq 1,\ b > 0$, and $x > 0$, then the following is true.

- Evaluate $\log_3 17$ to four decimal places.

Using the Change-of-Base Rule

Use the change-of-base rule to find each logarithm to four decimal places.

$\log_4 18$

$\log_5 21$

Using the Change-of-Base Rule in An Application

Use natural logarithms in the change-of-base rule and the function defined by

$$f(x) = 2014 + 384.7 \log_2 x$$

to estimate crude oil imports (in millions of barrels) into the United States in 2008. Compare this to the actual amount of 3571 million barrels. In the equation, $x = 1$ represents 1990.