

10.6 Exponential and Logarithmic Equations; Further Application

By: Cindy Alder



Objectives:

- Solve equations involving variables in the exponents.
- Solve equations involving logarithms.
- Solve applications of compound interest.
- Solve applications involving base e exponential growth and decay

Properties for Solving Exponential and Logarithmic Equations

- For all real numbers $b > 0, b \neq 1$, and any real number x and y :
 1. If $x = y$, then $b^x = b^y$.
 2. If $b^x = b^y$, then $x = y$.
 3. If $x = y$, and $x > 0, y > 0$, then $\log_b x = \log_b y$.
 4. If $x > 0, y > 0$, and $\log_b x = \log_b y$, then $x = y$.



Solving an Exponential Equation

Solve the equation. Approximate the solution to three decimal places.

$$2^x = 16$$

$$10^x = 4$$



Solving an Exponential Equation

Solve the equation. Approximate the solution to three decimal places.

$$6^{x+1} = 22$$



Solving an Exponential Equation

Solve the equation. Approximate the solution to three decimal places.

$$2^{x+3} = 5^x$$



Solving an Exponential Equation

Solve the equation. Approximate the solution to three decimal places.

$$e^{0.02x} = 192$$



Solving an Exponential Equation

Solve the equation. Approximate the solution to three decimal places.

$$e^{-0.103x} = 7$$



Solving a Logarithmic Equation

Solve the equation. Give the exact solution.

$$\log_3(x + 1)^5 = 3$$



Solving a Logarithmic Equation

Solve the equation. Give the exact solution.

$$\log_8(2x + 5) + \log_8 3 = \log_8 33$$



Solving a Logarithmic Equation

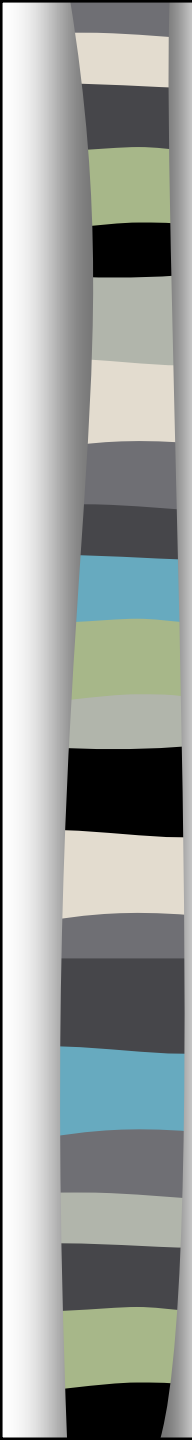
Solve $\log x + \log(x + 15) = 2$.

- We have solved simple interest problems using the formula $I = Prt$. In most cases, interest paid or charged is **compound interest** (interest paid on both principal and interest).

Compound Interest Formula (for a Finite Number of Periods)

If a principal of P dollars is deposited at an annual rate of interest r compounded (paid) n times per year, then the account will contain

dollars after t years. (In this formula, r is expressed as a decimal.)



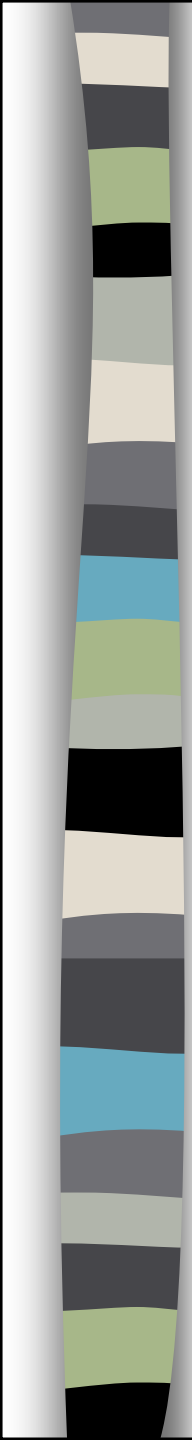
Solving a Compound Interest Problem for A .

Find the value of \$2000 deposited at 4% compounded quarterly for 10 years.



Solving a Compound Interest Problem for t .

Approximate the time it would take for money deposited in an account paying 4% compounded semiannually to double. Round to the nearest hundredth.

- 
- Interest can be compounded annually, semiannually, quarterly, daily, and so on. The number of compounding periods can get larger and larger. If the value of n is allowed to approach infinity, we have an example of **continuous compounding**.

Continuous Compound Interest Formula

If a principal of P dollars is deposited at an annual rate of interest r compounded **continuously** for t years, the final amount A on deposit is given by



Solving a Continuous Compound Interest Problem

Suppose that \$2000 is invested at 5% interest compounded continuously for 10 years.

- a) How much will the investment grow to if it is compounded continuously?



Solving a Continuous Compound Interest Problem

Suppose that \$2000 is invested at 5% interest compounded continuously for 10 years.

- b) Approximate the time it would take for the amount to double. Round to the nearest tenth.



Solving an Application Involving Exponential Decay

Radioactive strontium decays according to the function defined by

$$y = y_0 e^{-0.0239t}$$

where t is time in years.

- a) If an initial sample contains $y_0 = 12$ g of radioactive strontium, how many grams, to the nearest tenth, will be present after 35 years?



Solving an Application Involving Exponential Decay

Radioactive strontium decays according to the function defined by

$$y = y_0 e^{-0.0239t}$$

where t is time in years.

- b) Approximate the half-life of radioactive strontium. Round to the nearest unit.