# 10.6 Exponential and Logarithmic Equations; Further Application

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## Objectives:

- Solve equations involving variables in the exponents.
- Solve equations involving logarithms.
- Solve applications of compound interest.
- Solve applications involving base $e$ exponential growth and decay

# Properties for Solving Exponential and Logarithmic Equations

- For all real numbers $b > 0, b \neq 1$, and any real number $x$ and $y$:

1. If $x = y$, then $b^x = b^y$.
2. If $b^x = b^y$, then $x = y$.
3. If $x = y$, and $x > 0$, $y > 0$, then $\log_b x = \log_b y$.
4. If $x > 0$, $y > 0$, and $\log_b x = \log_b y$, then $x = y$.

# Solving an Exponential Equation

**Solve the equation. Approximate the solution to three decimal places.**

$2^x = 16$

$10^x = 4$

# Solving an Exponential Equation

**Solve the equation. Approximate the solution to three decimal places.**

$$6^{x+1} = 22$$

# Solving an Exponential Equation

**Solve the equation. Approximate the solution to three decimal places.**

$$2^{x+3} = 5^{x}$$

# Solving an Exponential Equation

**Solve the equation. Approximate the solution to three decimal places.**

$$e^{0.02x} = 192$$

# Solving an Exponential Equation

**Solve the equation. Approximate the solution to three decimal places.**

$$e^{-0.103x} = 7$$

# Solving a Logarithmic Equation

**Solve the equation. Give the exact solution.**

$$\log_3(x+1)^5 = 3$$

# Solving a Logarithmic Equation

**Solve the equation. Give the exact solution.**

$$\log_8(2x + 5) + \log_8 3 = \log_8 33$$

# Solving a Logarithmic Equation

**Solve** $\log x + \log(x + 15) = 2$**.**

- We have solved simple interest problems using the formula $I = Prt$. In most cases, interest paid or charged is **compound interest** (interest paid on both principal and interest).

## Compound Interest Formula (for a Finite Number of Periods)

If a principal of $P$ dollars is deposited at an annual rate of interest $r$ compounded (paid) $n$ times per year, then the account will contain

dollars after $t$ years. (In this formula, $r$ is expressed as a decimal.)

# Solving a Compound Interest Problem for *A*.

Find the value of $2000 deposited at 4% compounded quarterly for 10 years.

# Solving a Compound Interest Problem for *t*.

Approximate the time it would take for money deposited in an account paying 4% compounded semiannually to double.  Round to the nearest hundredth.

- Interest can be compounded annually, semiannually, quarterly, daily, and so on. The number of compounding periods can get larger and larger. If the value of $n$ is allowed to approach infinity, we have an example of **continuous compounding.**

## Continuous Compound Interest Formula

If a principal of $P$ dollars is deposited at an annual rate of interest $r$ compounded **continuously** for $t$ years, the final amount $A$ on deposit is given by

# Solving a Continuous Compound Interest Problem

Suppose that $2000 is invested at 5% interest compounded continuously for 10 years.

a) How much will the investment grow to if it is compounded continuously?

# Solving a Continuous Compound Interest Problem

Suppose that $2000 is invested at 5% interest compounded continuously for 10 years.

b) Approximate the time it would take for the amount to double. Round to the nearest tenth.

# Solving an Application Involving Exponential Decay

Radioactive strontium decays according to the function defined by

$$y = y_0 e^{-0.0239t}$$

where $t$ is time in years.

a) If an initial sample contains $y_0 = 12$ g of radioactive strontium, how many grams, to the nearest tenth, will be present after 35 years?

# Solving an Application Involving Exponential Decay

Radioactive strontium decays according to the function defined by

$$y = y_0 e^{-0.0239t}$$

where $t$ is time in years.

b) Approximate the half-life of radioactive strontium. Round to the nearest unit.