# 11.2 Circles & Ellipses

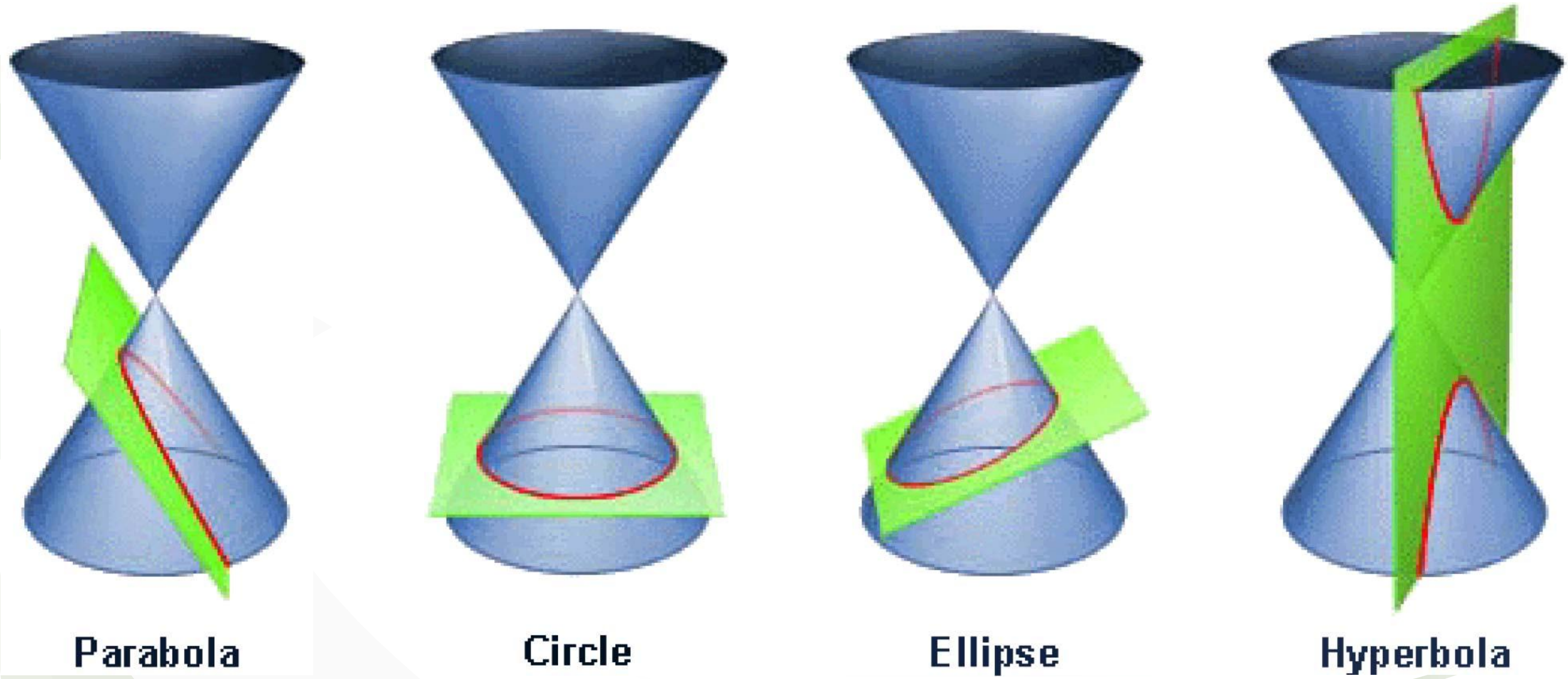
## OBJECTIVES:

- Find an equation of a circle given the center and the radius.
- Determine the center and radius of a circle given its equation.
- Recognize an equation of an ellipse.
- Graph ellipses.

Written by: Cindy Alder

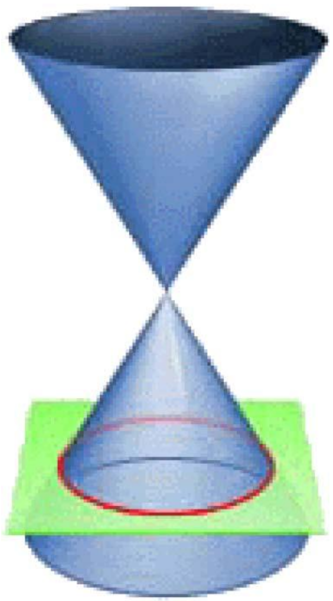
# What are Conic Sections?

- **Conic Sections** are curves obtained by intersecting a right circular cone with a plane.

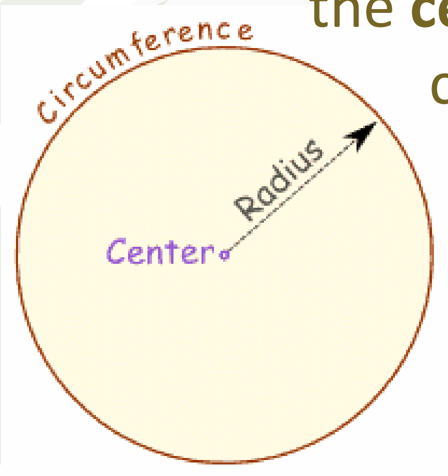


# The Circle

- A ***circle*** is formed when a plane cuts the cone at right angles to its axis.




- The definition of a **circle** is the set of all points in a plane such that each point in the set is equidistant from a fixed point called the **center**. The distance from the center is called the **radius.** The distance around the circle is called the **circumference.**

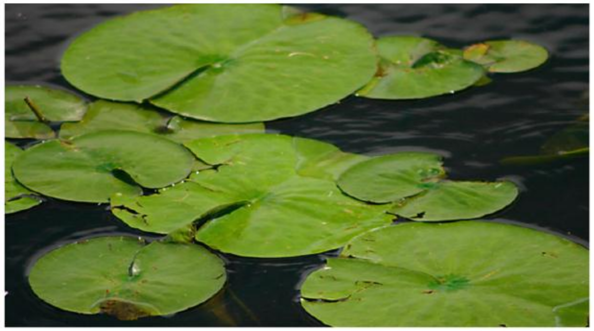


# Circles in Nature

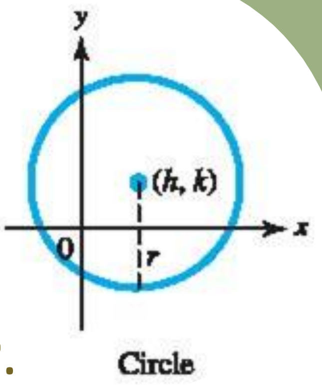
- Circles can also be seen all around us in nature.  One example can be seen with the armillaria mushrooms clustered around a stump.

- Another example is that of the stone circles of nature that cover the ground in parts of Alaska and the Norwegian island of Spitsbergen According to scientists, the circles are due to cyclic freezing and thawing of the ground that drives a simple feedback mechanism that generates the patterns.

# Circles in Nature



# Equation of a Circle



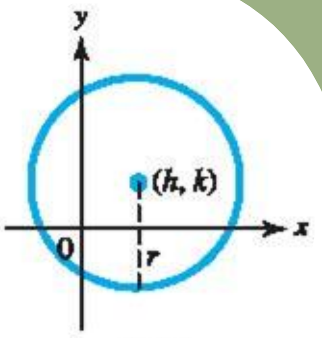

- Recall that the definition of a **circle** is the set of all points in a plane such that each point in the set is equidistant from a fixed point called the **center**.
- The equation of a circle is found using the distance formula:

$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

- A circle with center $(h, k)$ and radius $r > 0$ passing through the point $(x, y)$ must satisfy the equation:

# Equation of a Circle

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$




Circle

- Squaring both sides of this equation gives the **standard form** for the equation of a circle:

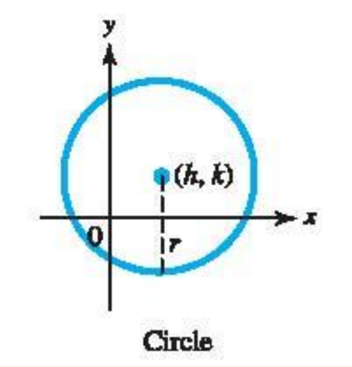
# Finding an Equation of a Circle and Graphing It.

- Find an equation of the circle with center at ( 0, 0 ) and radius 4. Then graph the circle.

# Finding an Equation of a Circle and Graphing It.

- Find an equation of the circle with center at ( 2, -1 ) and radius 3. Then graph the circle.

# Equation of a Circle

| EQUATION | GRAPH | DESCRIPTION | IDENTIFICATION |
| --- | --- | --- | --- |
| $(x-h)^2+(y-k)^2=r^2$ |  | The center is $(h,k)$ and the radius is r. | $x^2$ and $y^2$ terms have the same positive coefficients. |

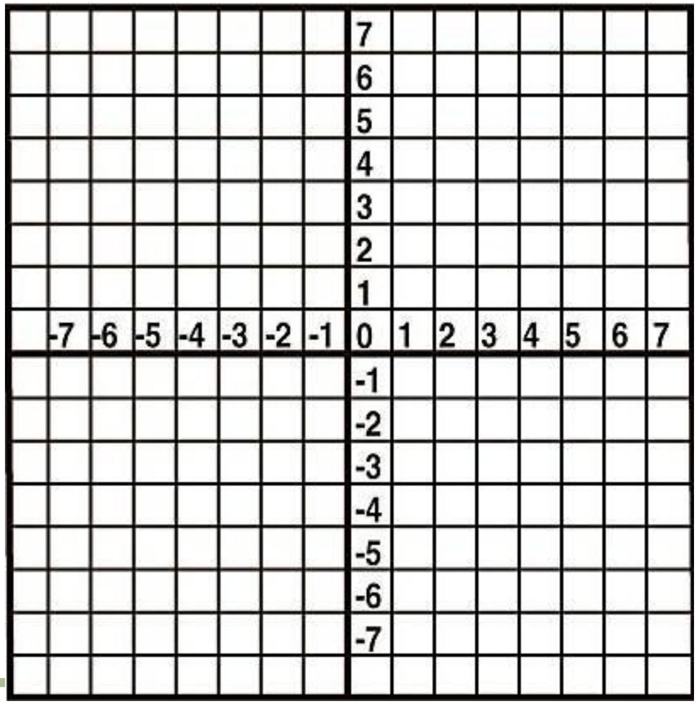
# Graphing a Circle

Graph the circle with equation $(x+2)^2 + (y-3)^2 = 25$

- Find the center $(h, k)$ and plot that point.

- Use the radius, $r$, to get four points on the circle.

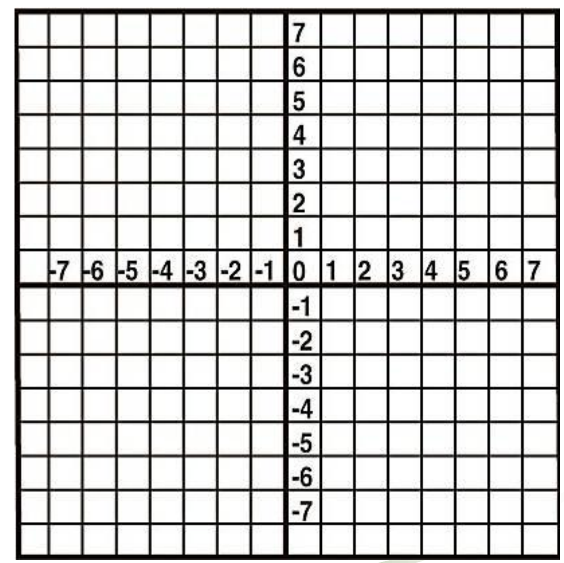
- Connect the points with a curve.



# Completing the Square to Find the Center and Radius of a Circle

- Find the center and radius of the circle. Then graph the circle.

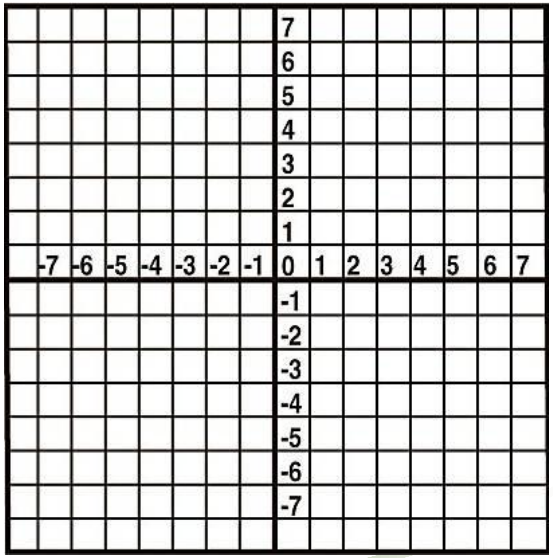
$$x^2 + y^2 + 6x - 4y - 51 = 0$$



# Completing the Square to Find the Center and Radius of a Circle

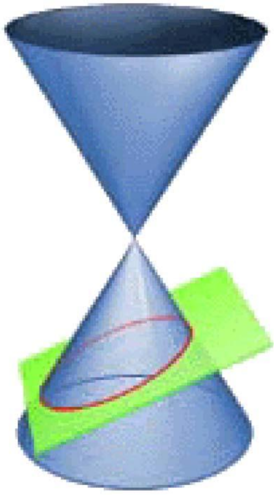
- Find the center and radius of the circle.  Then graph the circle.

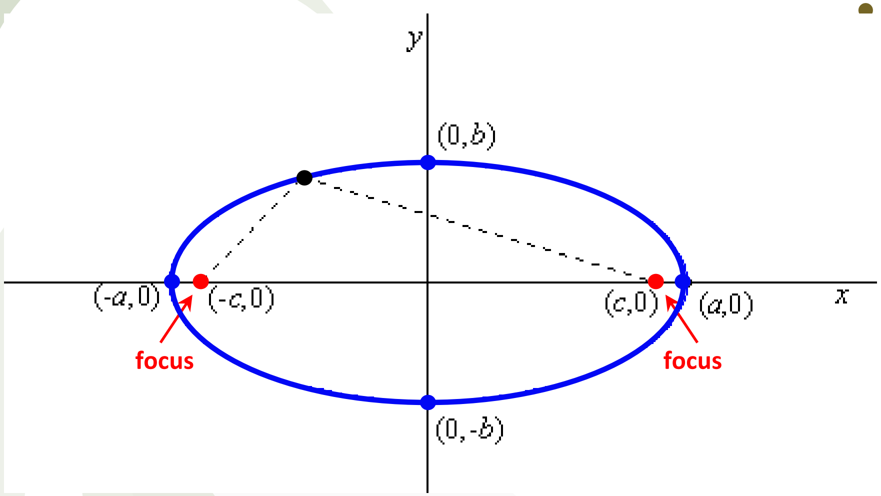
$$x^2 + y^2 + 2x + 6y - 15 = 0$$



# The Ellipse

- An ***ellipse*** is formed when a plane cuts the cone at an angle between a perpendicular to the axis (which would produce a circle), and an angle parallel to the side of the cone (which would produce a parabola).




- The definition of an **ellipse** is the set of all points in a plane, the sum of whose distances from two fixed points, called ***foci***, is a constant. ("Foci" is the plural of "focus" and is pronounced FOH-sigh.) The **origin** is the center of the ellipse. (A circle is a special case of an ellipse, where $a^2 = b^2$)

# Why are the Foci of an Ellipse Important?

- The ellipse has an important property that is used in the reflection of light and sound waves. Any light or signal that starts at one focus will be reflected to the other focus. This principle is used in *lithotripsy*, a medical procedure for treating kidney stones. The patient is placed in a elliptical tank of water, with the kidney stone at one focus. High-energy shock waves generated at the other focus are concentrated on the stone, pulverizing it.

# Ellipses in Physical Situations

Statuary Hall in the U.S. Capital building is elliptic. It was in this room that John Quincy Adams, while a member of the House of Representatives, discovered this acoustical phenomenon. He situated his desk at a focal point of the elliptical ceiling, easily eavesdropping on the private conversations of other House members located near the other focal point.

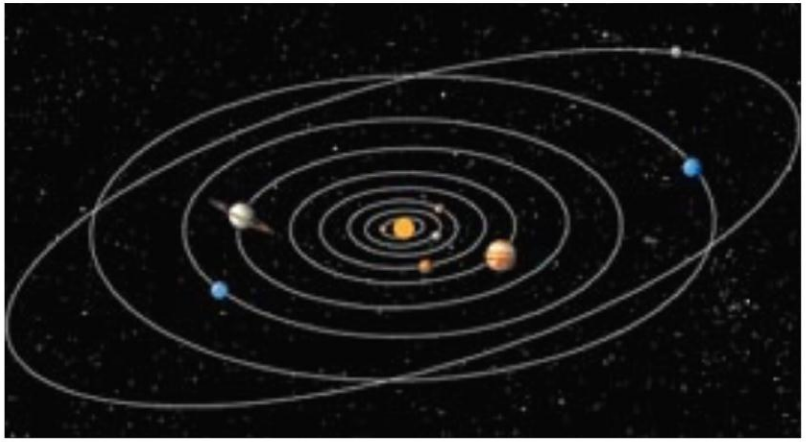
# Ellipses in Physical Situations

St. Paul's Cathedral in London. If a person whispers near one focus, he can be heard at the other focus, although he cannot be heard at many places in between.

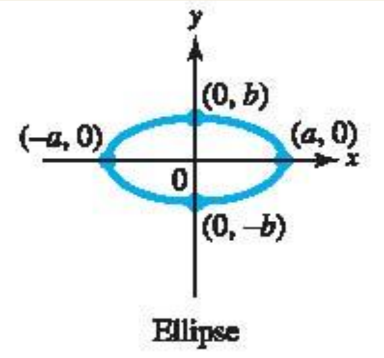
# Ellipses in Physical Situations

- Any cylinder sliced on an angle will reveal an ellipse in cross-section (as seen in the Tycho Brahe Planetarium in Copenhagen).

- The early Greek astronomers thought that the planets moved in circular orbits about an unmoving earth, since the circle is the simplest mathematical curve. In the 17th century, Johannes Kepler eventually discovered that each planet travels around the sun in an elliptical orbit with the sun at one of its foci.



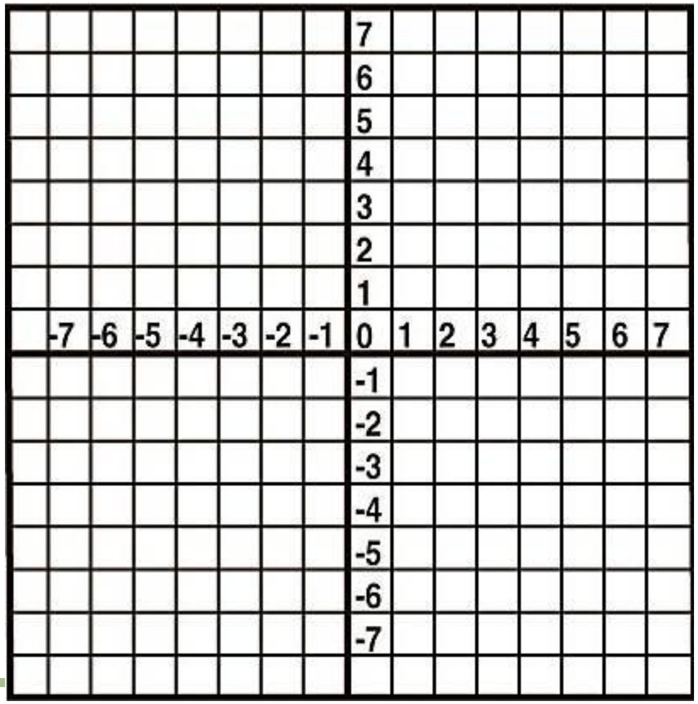
# Equation of an Ellipse Centered at the Origin

| Equation | Graph | Description | Identification |
|---|---|---|---|
| $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ |  | The $x$-intercepts are $(a, 0)$ and $(-a, 0)$. The $y$-intercepts are $(0, b)$ and $(0, -b)$. | $x^2$ and $y^2$ terms have different positive coefficients. |

# Graphing an Ellipse

Graph the ellipse with equation $$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

- Find the $x$- and $y$- intercepts.

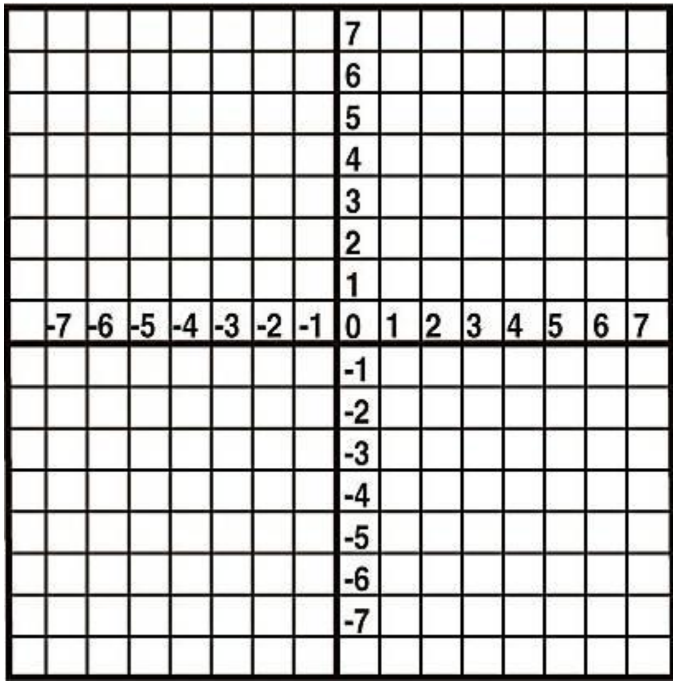



- Plot the $x$- and $y$- intercepts.

- Connect the points with an egg-shaped curve.

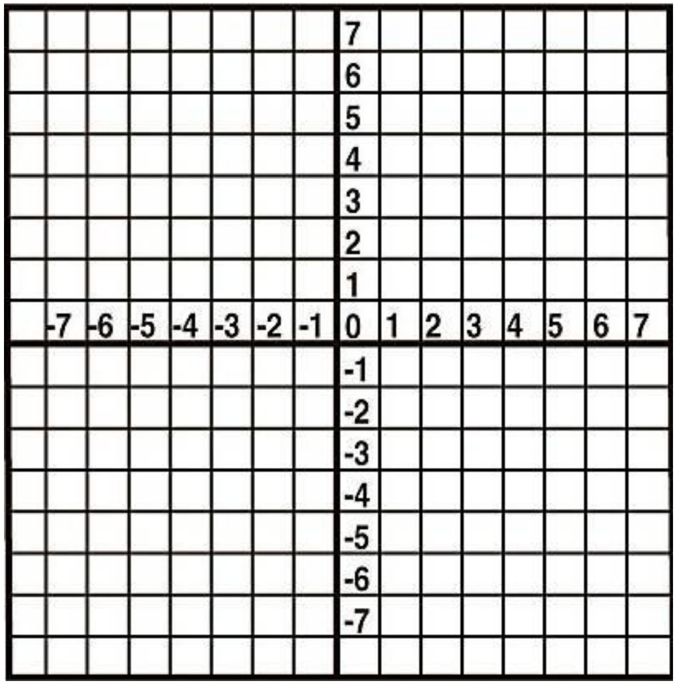
# Graphing Ellipses

- Graph: $$\frac{x^2}{49}+\frac{y^2}{36}=1$$



# Graphing Ellipses

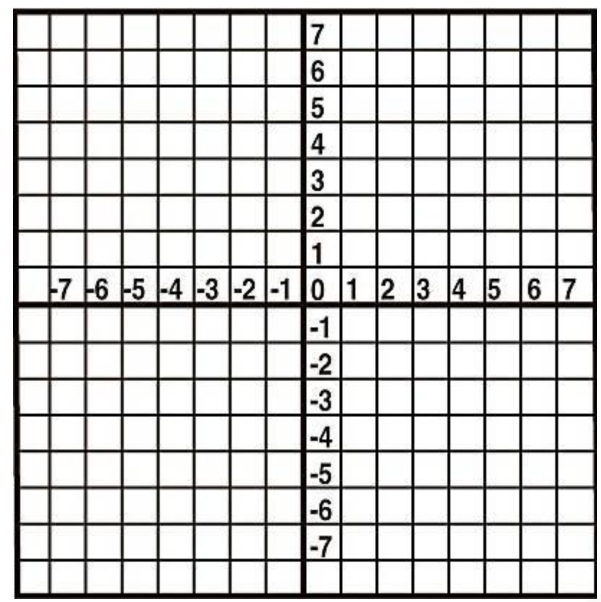
- Graph: $4x^2 + 25y^2 = 100$



# Graphing Ellipses

- Graph: $$\frac{(x-2)^2}{25}+\frac{(y+3)^2}{49}=1$$

(Hint: Use horizontal and vertical shifts like we did with parabolas.)



# Graphing Ellipses

- Graph: $\dfrac{(x+1)^2}{9}+\left(y-5\right)^2=1$

(Hint: Use horizontal and vertical shifts like we did with parabolas.)