11.3 The Hyperbola

OBJECTIVES:

- Recognize the equation of a hyperbola.
- Graph hyperbolas by using asymptotes.
- Identify conic sections by their equations.

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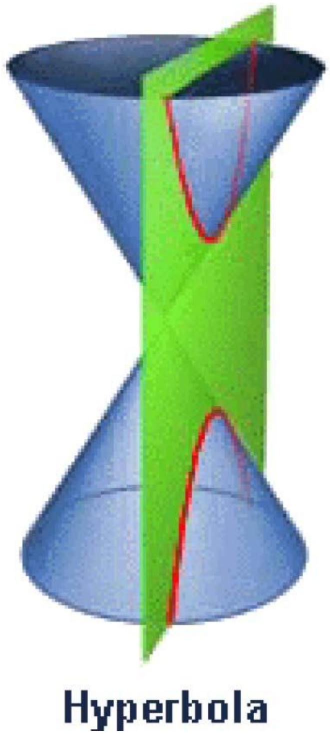
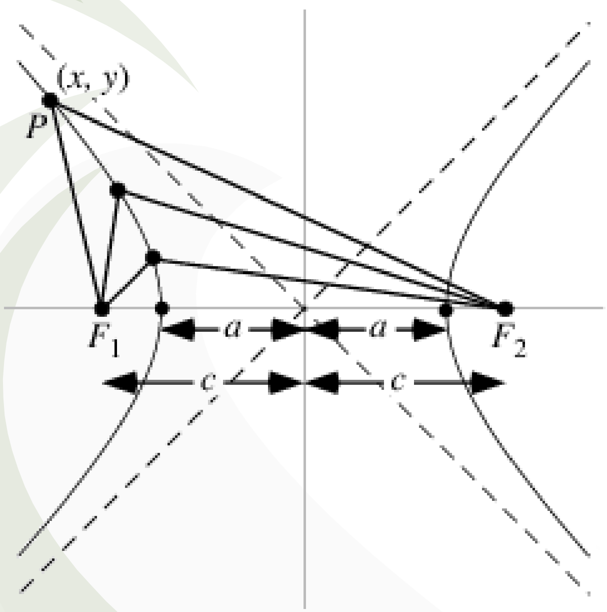
The Hyperbola

- A ***hyperbola*** is formed when a plane cuts the cone at an angle closer to the axis than the side of the cone.



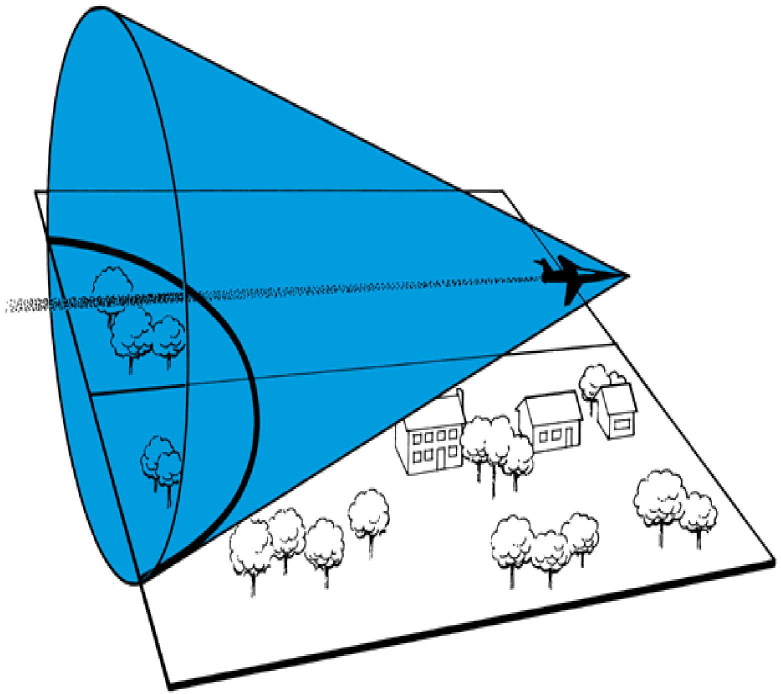
- The definition of a **hyperbola** is the set of all points in a plane, the difference of whose distances from two fixed points, called the **foci**, remains constant.



Hyperbola in Physical Situations

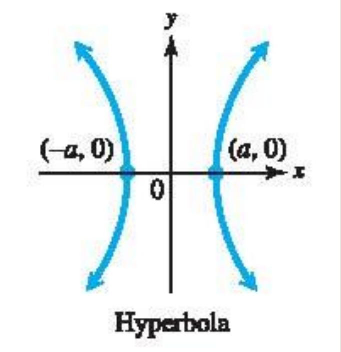
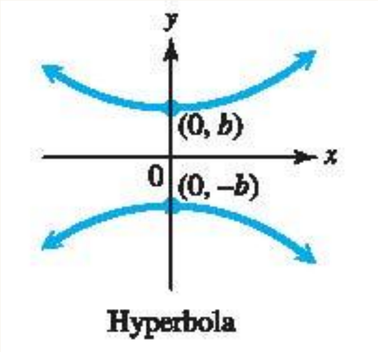
A hyperbola revolving around its axis forms a surface called a **hyperboloid**. The cooling tower of a steam power plant has the shape of a hyperboloid, as does the architecture of the James S. McDonnell Planetarium of the St. Louis Science Center.

Hyperbolas in Physical Situations



- A sonic boom shock wave has the shape of a cone, and it intersects the ground in part of a hyperbola. It hits every point on this curve at the same time, so that people in different places along the curve on the ground hear it at the same time. Because the airplane is moving forward, the hyperbolic curve moves forward and eventually the boom can be heard by everyone in its path.

Equation of a Hyperbola Centered at the Origin

Equation	Graph	Description	Identification
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$		The x-intercepts are $(a, 0)$ and $(-a. 0)$. The asymptotes are found from (a, b), $(a, -b), (-a, -b)$, and $(-a, b)$.	x^2 has a positive coefficient. y^2 has a negative coefficient.
$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$		The y-intercepts are $(0, b)$ and $(0, -b)$. The asymptotes are found from (a, b), $(a, -b), (-a, -b)$, and $(-a, b)$.	y^2 has a positive coefficient. x^2 has a negative coefficient.

Graphing Hyperbolas

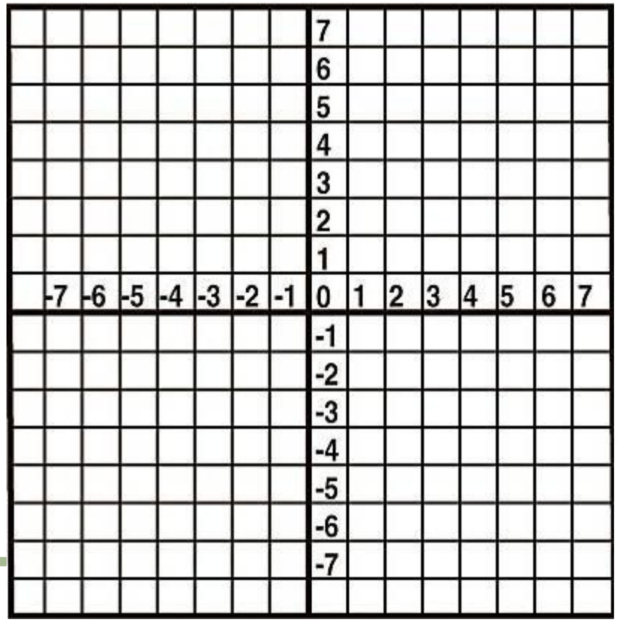
$$\frac{y^2}{16}-\frac{x^2}{9}=1$$

To graph a hyperbola:

- Find the values of a and b.
- Find and sketch the x or y-intercepts. (x-intercepts if x^2 has a positive coefficient, y-intercepts if y^2 has a positive coefficient)
- Find and sketch the fundamental rectangle. The vertices are located at $(a,b),(-a,b),(-a,-b),\ and$ $(a,-b)$.
- Sketch the asymptotes. (the extended diagonals of the rectangle with the following equations: $y=\pm\frac{b}{a}x$)
- Draw the graph.

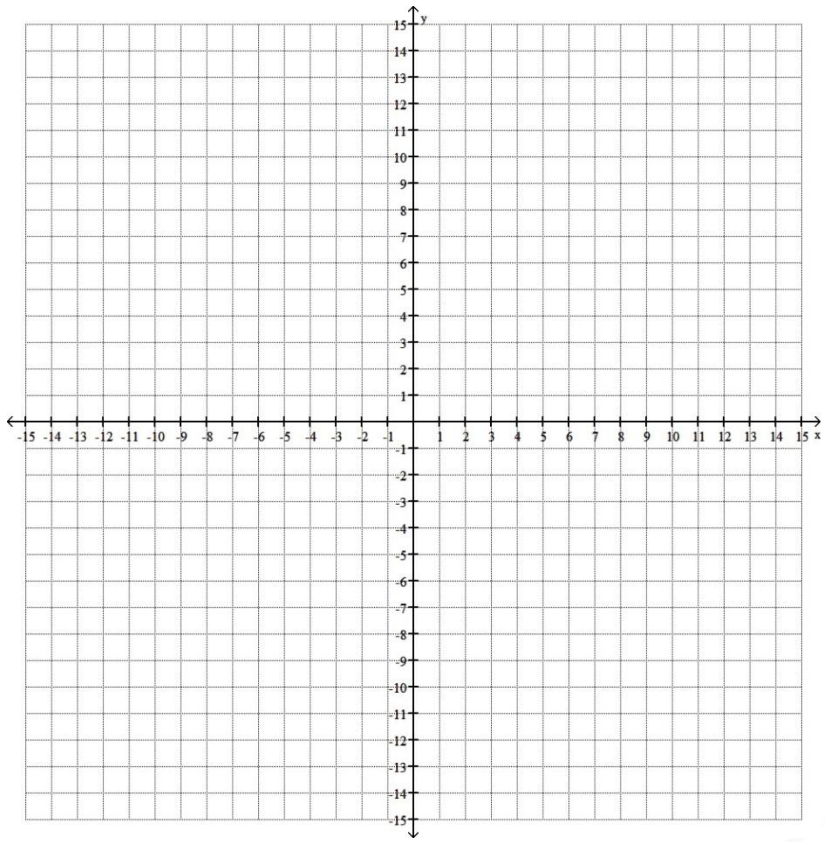
$y\text{-}intercpets:(0,4)\ and\ (0,-4)$

$vertices:(3,4),(-3,4),(-3,-4)\ and\ (3,-4)$



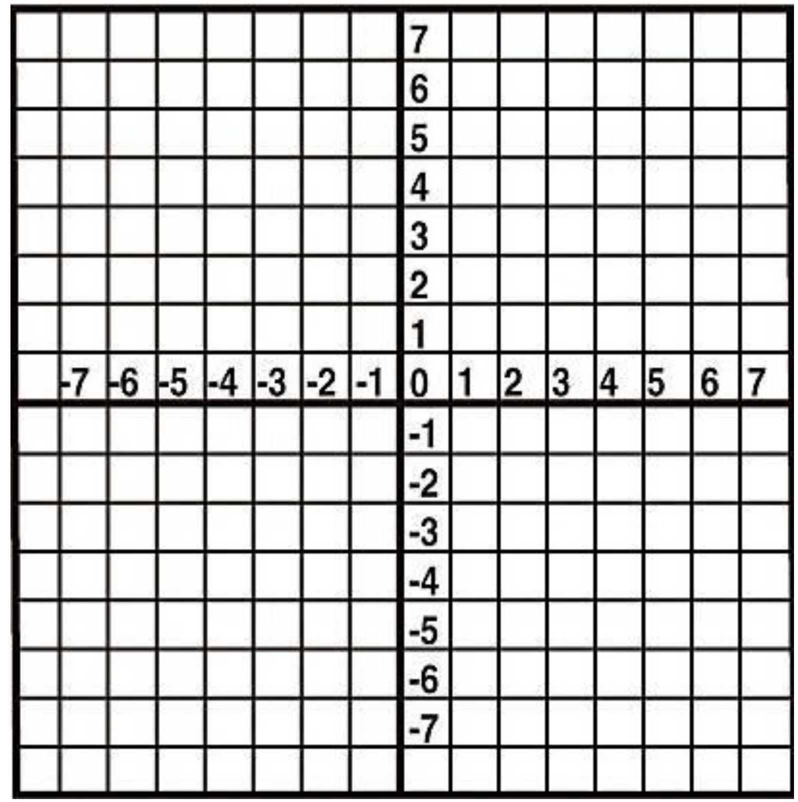
Graphing Hyperbolas

- Graph: $$\frac{x^2}{81} - \frac{y^2}{64} = 1$$



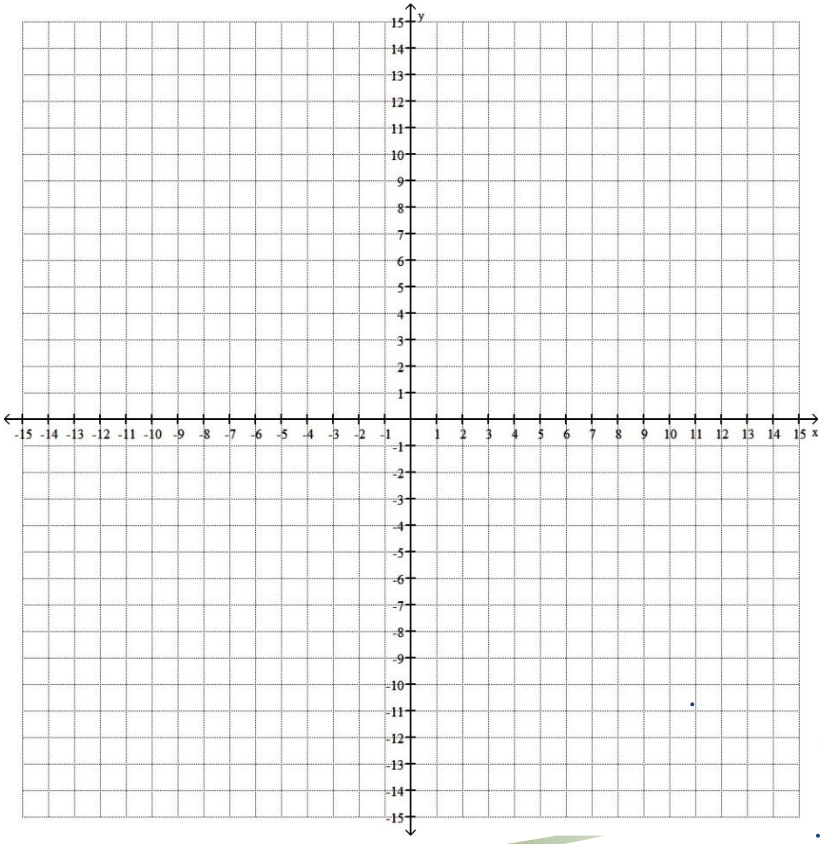
Graphing Hyperbolas

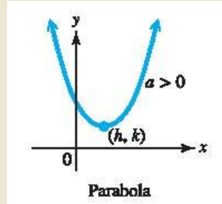
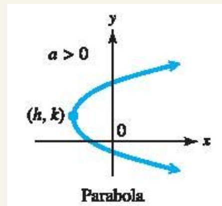
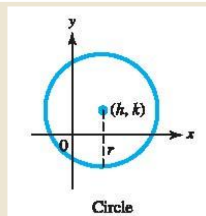
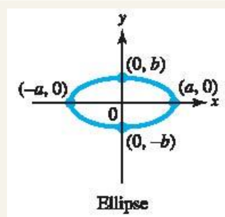
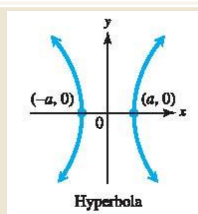
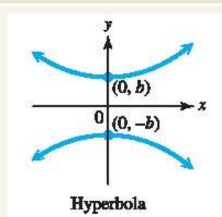
- Graph: $35y^2 - 4x^2 = 140$



Graphing Hyperbolas

- Graph: $\frac{(x+3)^2}{16} - \frac{(y-2)^2}{25} = 1$



Equation	Graph	Description	Identification
$y = a(x-h)^2 + k$		Parabola opens up if $a > 0$, down if $a < 0$. The vertex is (*h, k*). The axis is $x = h$.	It has an x^2 term and y is not squared.
$x = a(y-k)^2 + h$		Parabola opens to the right if $a > 0$, to the left if $a < 0$. The vertex is (*h, k*). The axis is $y = k$.	It has a y^2 term and x is not squared.
$(x-h)^2 + (y-k)^2 = r^2$		The center is (*h, k*) and the radius is r.	x^2 and y^2 terms have the same positive coefficients.
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$		The x-intercepts are (*a*, 0) and (-*a*, 0). The y-intercepts are (0, *b*) and (0, -*b*).	x^2 and y^2 terms have different positive coefficients.
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$		The x-intercepts are (*a*, 0) and (-*a*, 0). The asymptotes are found from (*a, b*), (*a, -b*), (*-a, -b*), and (*-a, b*).	x^2 has a positive coefficient. y^2 has a negative coefficient.
$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$		The y-intercepts are (0, *b*) and (0, -*b*). The asymptotes are found from (*a, b*), (*a, -b*), (*-a, -b*), and (*-a, b*).	y^2 has a positive coefficient. x^2 has a negative coefficient.

Identifying Conic Sections from Their Equations

- **Identify the graph of each equation.**
  - $9x^2 = 108 + 12y^2$
  - $3x^2 = 27 - 4y^2$
  - $x^2 = y - 3$
  - $6x^2 = 100 + 2y^2$
  - $x^2 = 9 - y^2$
  - $3x^2 = 27 - 4y$

References

- Britton, Jill. Occurrence of the Conics. September 25, 2003. June 3O, 2004 http://britton.disted.camosun.bc.ca/jbconics.htm
- Wolfram Mathworld. Conic Sections http://mathworld.wolfram.com/Hyperbola.html
- Courtney Seligman, Professor of Astronomy / Author, Ellipses and Other Conic Sections . http://cseligman.com/text/history/ellipses.htm
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- Lial, Hornsby, & McGinnis. Beginning and Intermediate Algebra. 2008