9.6 More About Parabolas & Their Applications

OBJECTIVES:

- Find the vertex of the vertical parabola.
- Graph a quadratic function.
- Use the discriminant to find the number of *x*-intercepts of a parabola with a vertical axis.
- Use quadratic functions to solve problems involving maximum or minimum values.
- Graph parabolas with horizontal axes.

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Quadratic Equations

- We can tell a lot about a quadratic equation when it is in the form:

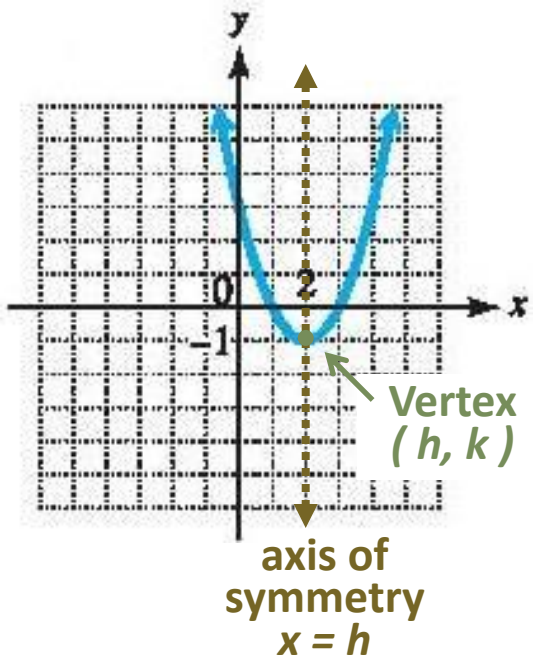
$$f(x) = a(x - h)^2 + k$$

- Vertex:
- Axis of Symmetry:
- Direction of opening: **UP** if

  DOWN if
- Wide/Narrow:

  WIDE if

  NARROW if
- Vertical Shift:
- Horizontal Shift:



Quadratic Equations

- What do we do when the quadratic equation is in the form:

$$f(x) = ax^2 + bx + c$$

COMPLETE THE SQUARE!!!!

Complete the Square when $a = 1$

$$f(x) = x^2 - 4x + 5$$

- Group the variable terms. Slide the constant over to "deal with later".
- Find the "magic" number needed to complete the square.
- Add and subtract the "magic" number to the equation.
- Factor and combine like terms.

Find the Vertex of the Graph of

$$f(x) = x^2 + 6x + 14$$

Find the Vertex of the Graph of

$$f(x) = x^2 - x + 7$$

Complete the Square when $a \neq 1$

$$f(x) = -3x^2 + 6x - 1$$

- Group the variable terms. Slide the constant over to "deal with later".
- Factor out the coefficient on x^2 from the variable terms.
- Find the "magic" number needed to complete the square.
- Add the "magic" number to the equation and
- Subtract the product of the "magic" number and the coefficient you factored out from the equation.
- Factor and combine like terms.

Find the Vertex of the Graph of

$$f(x) = 4x^2 - 32x + 58$$

Find the Vertex of the Graph of

$$f(x) = 2x^2 - 4x + 1$$

The Vertex Formula

$$f(x) = ax^2 + bx + c$$

Vertex Formula

$$\left(\frac{-b}{2a}, f\left(\frac{-\mathrm{b}}{2\mathrm{a}}\right)\right)$$

$$x = \frac{-b}{2a} \qquad y = f\,(the\ value\ you\ got\ for\ x)$$

Find the vertex using the vertex formula.

- $f(x) = x^2 + 8x + 10$

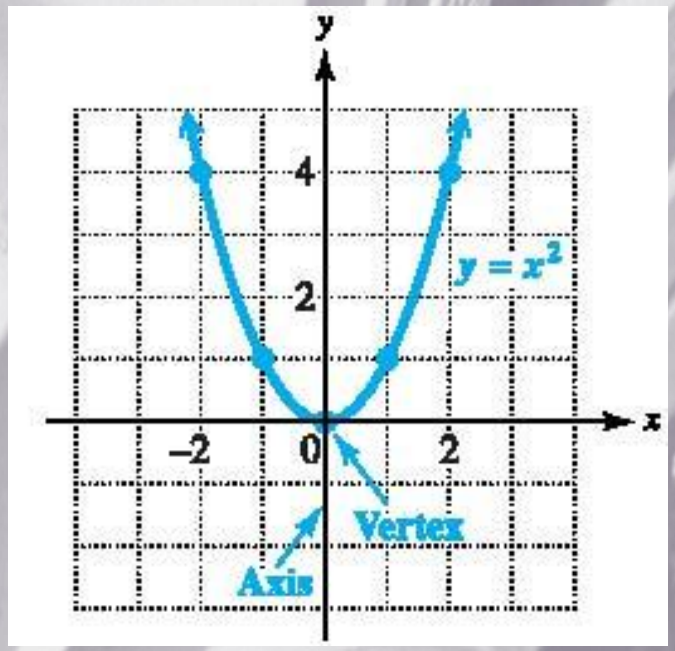
Vertex Formula

Find the vertex using the vertex formula.

- $f(x) = -2x^2 + 3x - 1$

Graphing a Quadratic Function

- **STEPS**
  - **Does the graph open up or down?**
  - **Find the vertex.**
    - Vertex formula
    - Complete the square
  - **Find any intercepts.**
    - *y*-intercept: set $x = 0$
    - *x*-intercept: set $y = 0$
  - **Complete the graph.**
    - Plot points found.
    - Plot additional points as needed, using symmetry.



A minimum of two points on either side of the vertex is needed for an accurate graph.

Graph the Quadratic Equation:

$$f(x) = x^2 - 6x + 5$$

- Step 1 - **Does the graph open up or down?**
  - The graph will open up because *a* is positive.

- Step 2 - **Find the vertex.**

Graph the Quadratic Equation (continued)

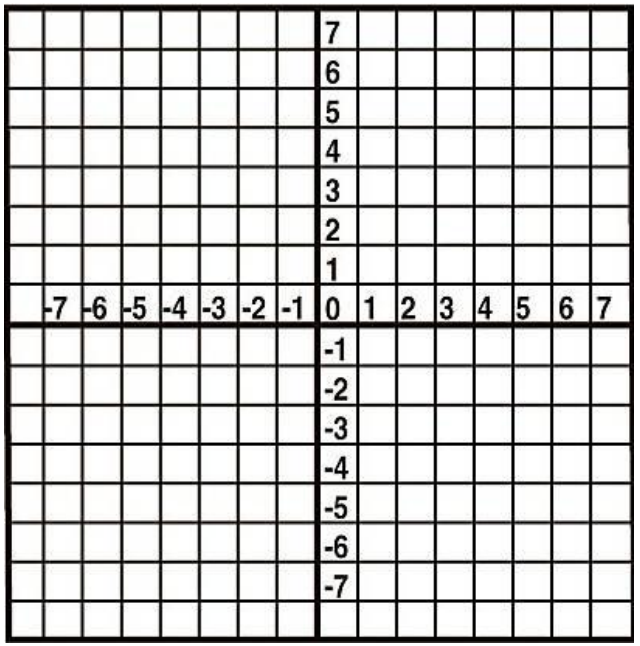
$$f(x) = x^2 - 6x + 5$$

- Step 3 - **Find any intercepts.**

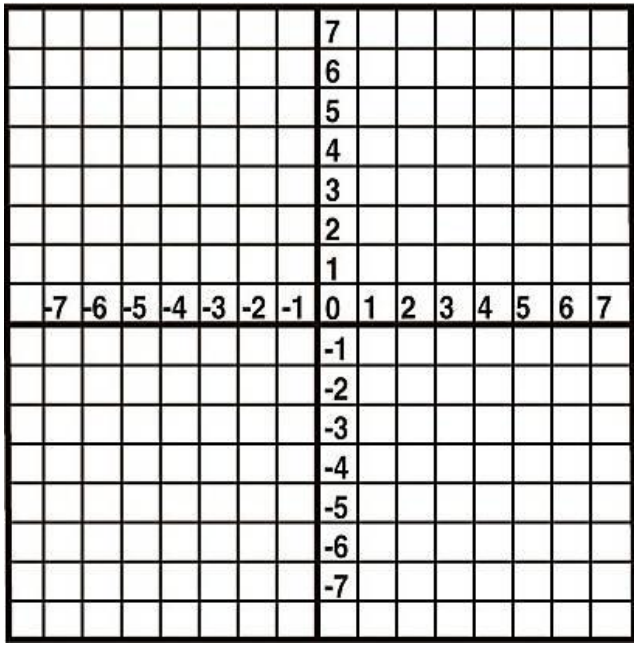
Graph the Quadratic Equation (continued)

$$f(x) = x^2 - 6x + 5$$

- Step 4 - **Complete the graph.**
  - Plot points found.
    - Vertex: $(3, -4)$
    - *y*-intercept: $(0,5)$
    - *x*-intercepts: $(5,0) and\ (1,0)$
  - Plot additional points as needed, using symmetry.
    - Using symmetry the graph will also pass through the point $(6,5)$.

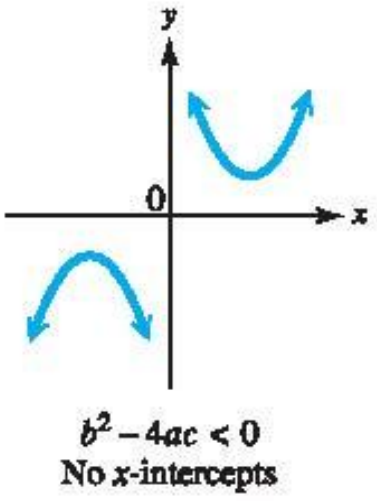


Graph the Function $f(x) = x^2 - x - 6$

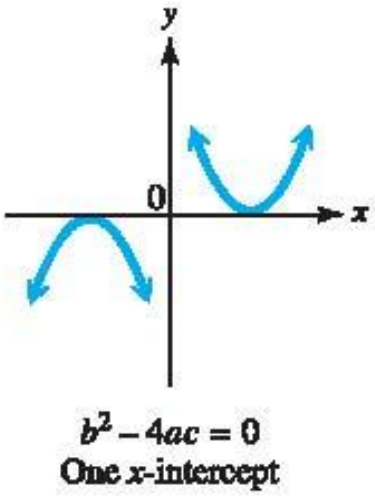


The Discriminant and x-intercepts.

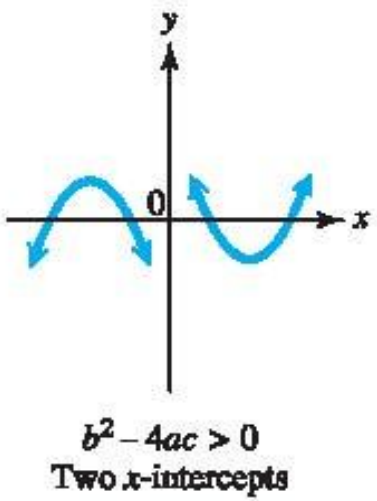
- The discriminant $b^2 - 4ac$, from the quadratic formula, can be used to determine the number of x-intercepts of the graph of a quadratic function.
  - If the discriminant is negative, the parabola will have no x-intercepts.
  - If the discriminant is 0, the parabola will have only one x-intercept.
  - If the discriminant is positive, the parabola will have two x-intercepts.



$b^2 - 4ac < 0$
No x-intercepts



$b^2 - 4ac = 0$
One x-intercept



$b^2 - 4ac > 0$
Two x-intercepts

Using the Discriminant

- Find the discriminant and use it to determine the number of *x*-intercepts of the graph of each quadratic function.

A) $f(x) = -3x^2 - x + 2$

B) $f(x) = x^2 - x + 1$

C) $f(x) = x^2 - 8x + 16$

Parabola with Horizontal Axis

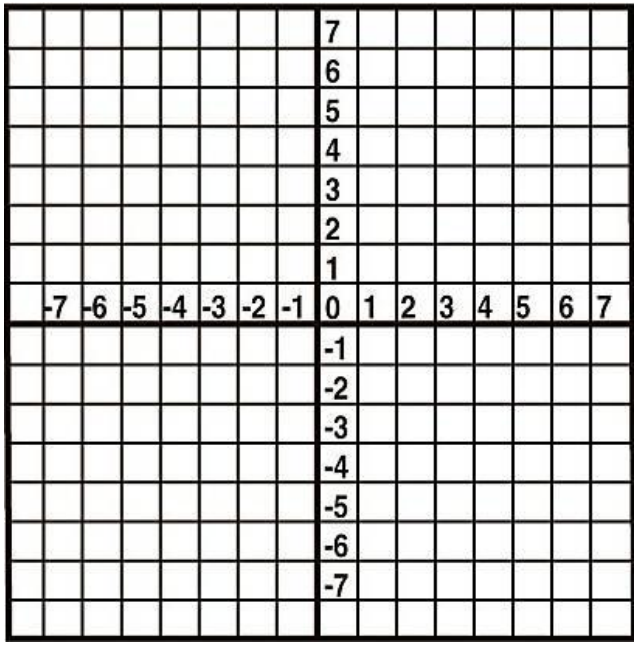
- The graph of

$$x = ay^2 + by + c \quad \text{or} \quad x = a(y-k)^2 + h$$

is a parabola with vertex (h, k) and the horizontal line $y = k$ as axis of symmetry. The graph opens to the right if $a > 0$ and to the left if $a < 0$.

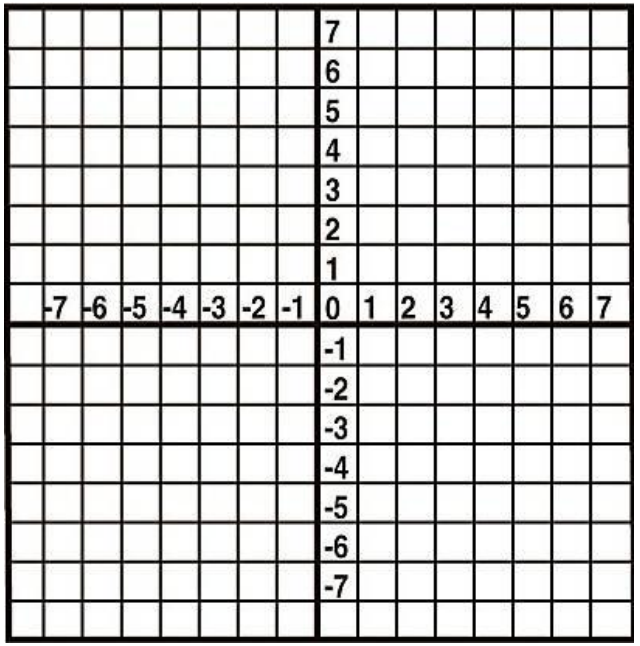
Graph the Parabola

- Graph $x = (y-2)^2 - 3$. Give the vertex, axis, domain, and range.
  - Identify h and k.



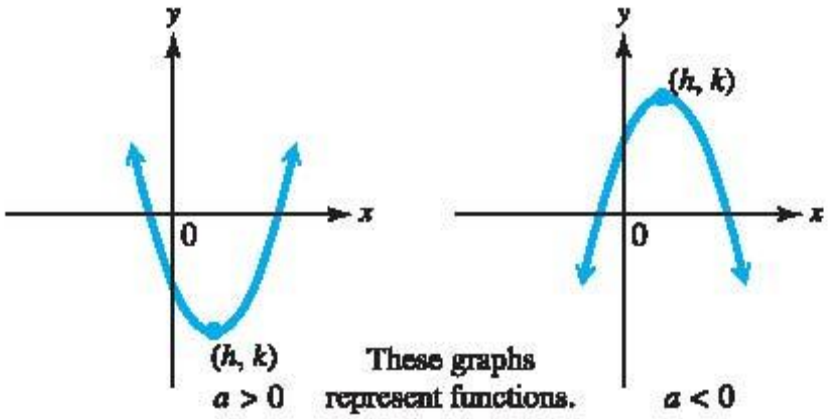
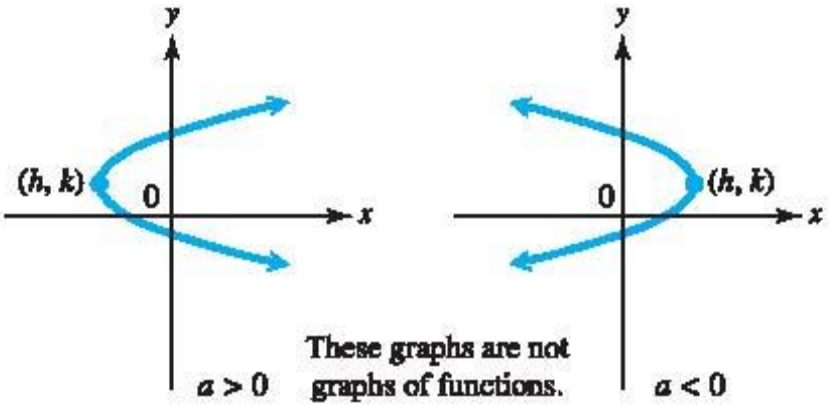
Graph the Parabola

- Graph $x = -y^2 + 2y + 5$. Give the vertex, axis, domain, and range.



Graphs of Parabolas

GRAPHS OF PARABOLAS

Equation	Graph
$y = ax^2 + bx + c$ $y = a(x - h)^2 + k$	
$x = ay^2 + by + c$ $x = a(y - k)^2 + h$	

Solving Maximum/Minimum Problems

- **General Concepts**
  - The vertex is either the **highest** or **lowest** points on a parabola depending on which way it opens.
  - The y-value of the vertex gives the maximum or minimum value of all y's and the x-value of the vertex tells where the maximum or minimum occurs.
  - In an applied problem, if you can create a quadratic function with the information given, you can then find the vertex to give you the **maximum or minimum y-value** and what **x-value is required to get the maximum or minimum.**

Finding the Maximum Area

- A farmer has 120 feet of fencing. He wants to put a fence around a rectangular field next to a building. Find the maximum area he can enclose, and the dimensions of the field when the area is maximized.

Finding the Maximum Height

- A toy rocket is launched from the ground so that its distance in feet above the ground after *t* seconds is

$$s(t) = -16t^2 + 208t$$

Find the maximum height it reaches and the number of seconds it takes to reach that height.