3.3 Linear Equations in Two Variables

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Objectives:

- Write an equation of a line, given its slope and *y*-intercept.
- Graph a line, using its slope and *y*-intercept.
- Write an equation of a line, given its slope and a point on the line.
- Write an equation of a line, given two points on the line.
- Write an equation of a line parallel or perpendicular to a given line.
- Write an equation of a line that models real data.

Slope-Intercept Form

The **slope-intercept form** of the equation of a line with slope m and y-intercept $(0, b)$ is

EXAMPLE 1

Identify the slope and y-intercept of the line with each equation.

a) $y = -4x + 1$

b) $y = x - 8$

c) $y = 6x$

d) $y = \frac{x}{4} - \frac{3}{4}$

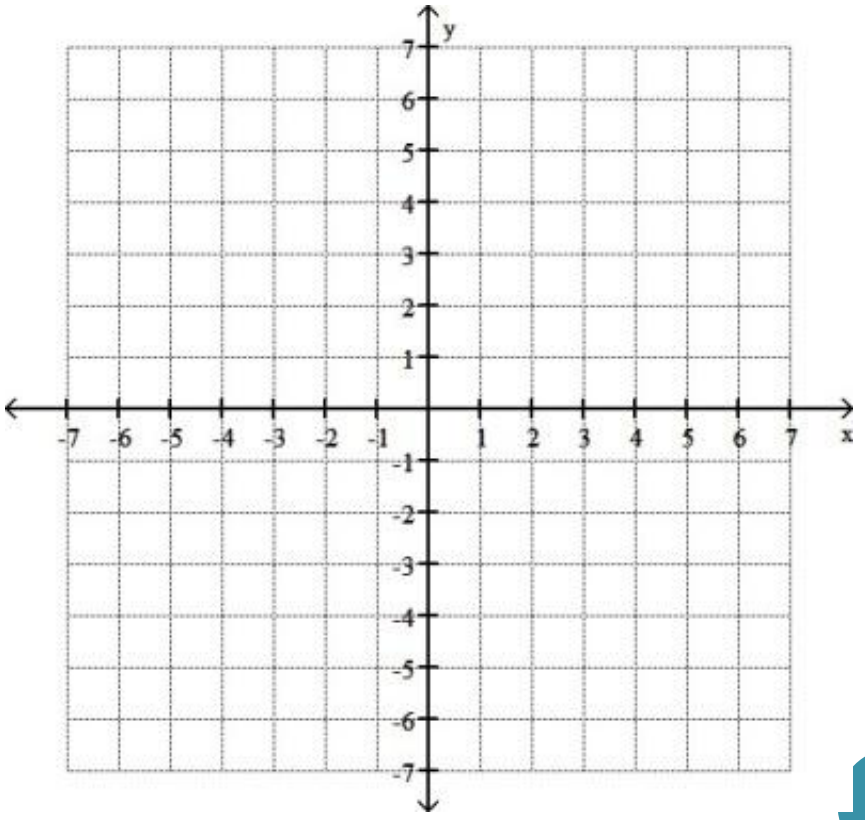
Example 2

- Write an equation of a line with slope of 2 and y-intercept $(0, -5)$.

- Write an equation of a line with slope of $-\frac{1}{3}$ and y-intercept $(0,7)$.

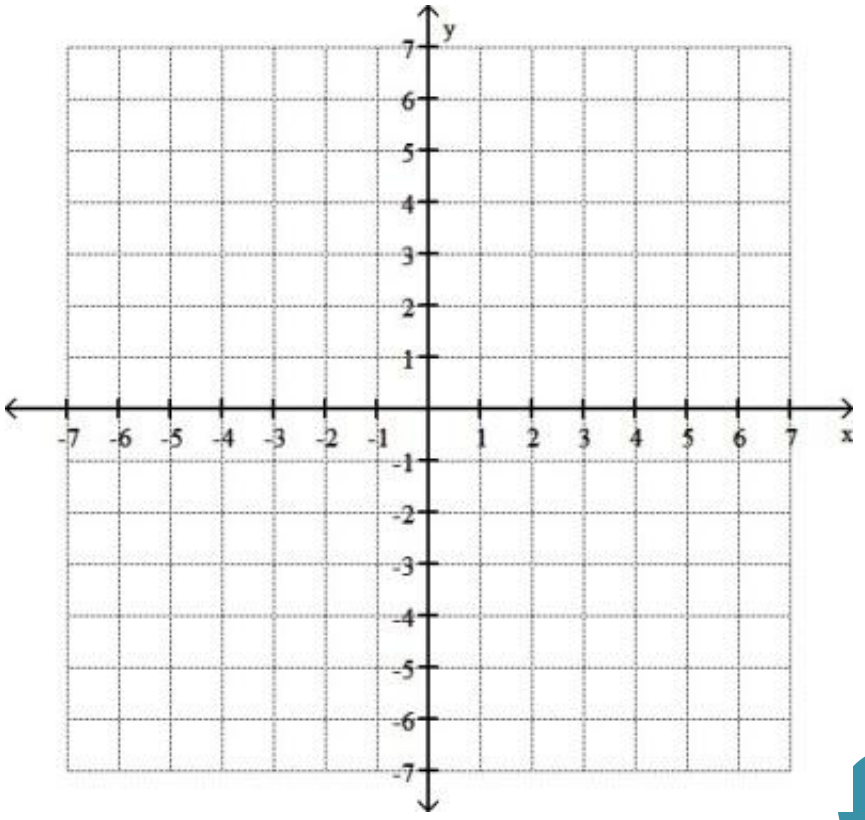
Example 3

- Graph the line $x + 2y = -4$ using the slope and y-intercept.



Example 4

- Graph the line $2x = 9 - 3y$ using the slope and y-intercept.



Point-Slope Form

The **point-slope form** of the equation of a line with slope m passing through (x_1, y_1) is

EXAMPLE 5

Write the equation of the line passing through $(-2,4)$ with slope of -3.

Example 6

- Write the equation of the line passing through (5,2) with slope of $-\frac{1}{3}$.

Equations of Horizontal and Vertical Lines

The **horizontal line** through $(0, b)$ has equation

The **vertical line** through $(a, 0)$ has equation

Example 7

Write the equation of the line with an undefined slope passing through (0,5).

Example 8

- Write the equation of the line passing through (0,5) with slope of 0.

- Write the equation of the line passing through (−2,7) with an undefined slope.

Example 9

- Write the equation of the line passing through the points $(-2,6)$ and $(1,4)$. Give the final answer in slope-intercept form and standard form.

Example 10

- Write the equation of the line passing through the points $(6, -9)$ and $(-3,5)$. Give the final answer in slope-intercept form and standard form.

Example 11

- Write an equation of a line passing through the point $(-8,3)$ and

  (a) parallel to the line $2x - 3y = 10$

  (b) perpendicular to the line $2x - 3y = 10$

a) $m_{\parallel} = \frac{2}{3}$ $(-8, 3)$

b) $m_{\perp} = -\frac{3}{2}$ $(-8, 3)$

Forms of Linear Equations

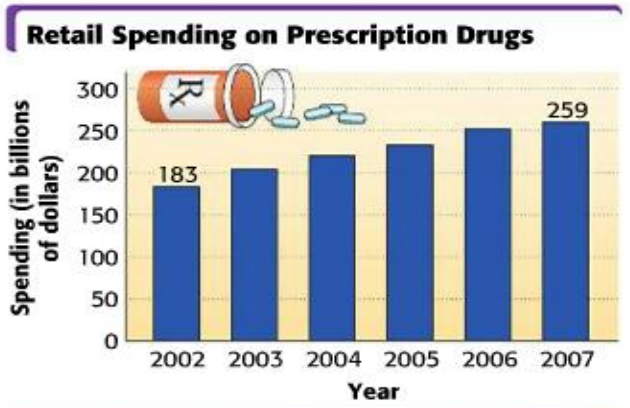
Equation	*Description*	*When to Use*
$y = mx + b$	**Slope-Intercept Form** Slope is m. y-intercept is $(0, b)$.	The slope and y-intercept can be easily identified and used to quickly graph the equation.
$y - y_1 = m(x - x_1)$	**Point-Slope Form** Slope is m. Line passes through (x_1, y_1).	This form is ideal for finding the equation of a line if the slope and a point on the line or two points on the line are known.
$Ax + By = C$	**Standard Form** (A, B, and C integers, $A \geq 0$) Slope is $-\frac{A}{B}$ $(B \neq 0)$. x-intercept is $\left(\frac{C}{A}, 0\right)$ $(A \neq 0)$. y-intercept is $\left(0, \frac{C}{B}\right)$ $(B \neq 0)$.	The x- and y-intercepts can be found quickly and used to graph the equation. The slope must be calculated.
$y = b$	**Horizontal Line** Slope is 0. y-intercept is $(0, b)$.	If the graph intersects only the y-axis, then y is the only variable in the equation.
$x = a$	**Vertical Line** Slope is undefined. x-intercept is $(a, 0)$.	If the graph intersects only the x-axis, then x is the only variable in the equation.

Example 12

- Suppose there is a flat rate of \$0.20 plus a charge of \$0.10 per minute to make a telephone call. Write an equation that gives the cost y in dollars for a call of x minutes.

EXAMPLE 13

- Retail spending (in billions of dollars) on prescription drugs in the United States is shown in the graph in Figure 34. Write an equation that models the data.



Source: National Association of Chain Drug Stores.

FIGURE 34