4.1 SYSTEMS OF LINEAR EQUATIONS IN TWO VARIABLES
(PART 2)
4.2 SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

OBJECTIVES:

- Solve linear systems (with two equations and two variables) by elimination.
- Solve special systems.
- Understand the geometry of systems of three equations in three variables.
- Solve linear systems (with three equations and three variables) by elimination.

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Solving a Linear System by Elimination

- **Step 1**
- **Step 2**
- **Step 3**
- **Step 4**
- **Step 5**
- **Step 6**

Solve the system by elimination.

$$
\begin{aligned}
3x - y &= 7 \\
2x + y &= 3
\end{aligned}
$$

Solve the system by elimination.

$$4x - 5y = -18$$
$$3x + 2y = -2$$

Solve the system by elimination.

$$3y = 8 + 4x$$

$$6x = 9 - 2y$$

Solve the system by elimination.

$$3x + y = -7$$
$$6x + 2y = 5$$

Solve the system by elimination.

$$\begin{aligned} 2x+5y&=1 \\ -4x-10y&=-2 \end{aligned}$$

Systems of Linear Equations in Three Variables

- A solution of an equation in three variables, such as

  is called an **ordered triple** and is written:

- For example, the ordered triple is a solution of the preceding equation, because

- The graph of an equation in three variables is a *plane*.

Graphs of Linear Equations in Three Variables

- Consider the solution of a system such as

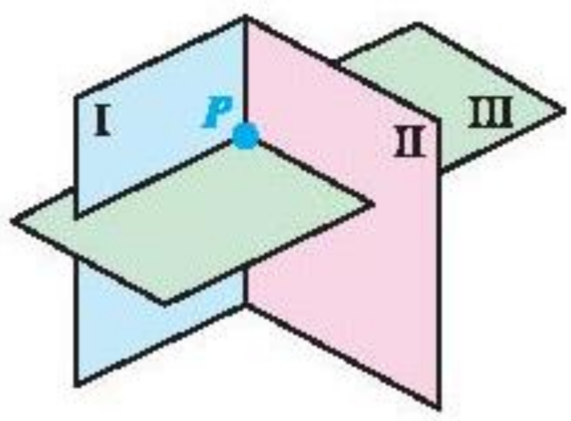
$$4x + 8y + z = 2$$
$$x + 7y - 3z = -14$$
$$2x - 3y + 2z = 3$$

- Because the graph of a linear equation in three variables requires three-dimensional graphing, it is not practical to solve these systems by graphing.

Graphs of Linear Equations in Three Variables

To help us visualize the possible solutions of such systems, we will examine the following graphs.

1. **The three planes may meet at a single, common point** that is the solution of the system.

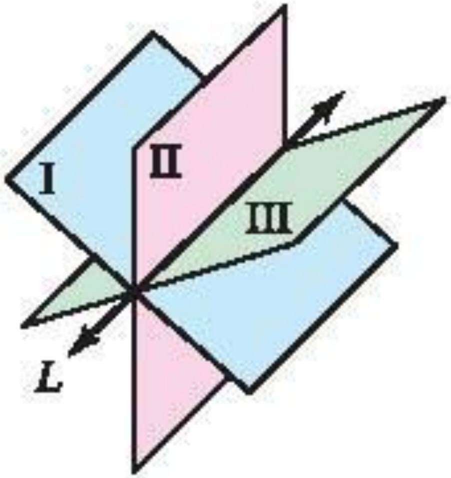


A single solution

(a)

Graphs of Linear Equations in Three Variables

2. **The three planes may have the points of a line in common,** so that the infinite set of points that satisfy the equation of the line is the solution of the system.

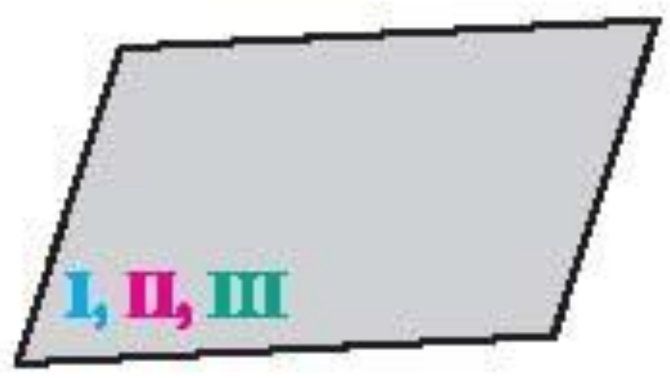


Points of a line in common

(b)

Graphs of Linear Equations in Three Variables

3. **The three planes may coincide,** so that the solution of the systems is the set of all points on a plane.

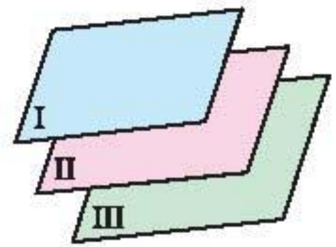


All points in common

(c)

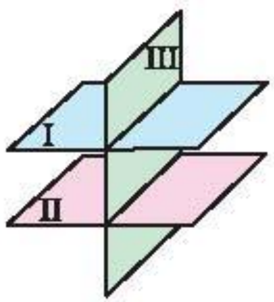
Graphs of Linear Equations in Three Variables

4. **The planes may have no points common to all three,** so there is no solution of the system.



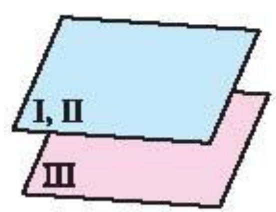
No points in common

(d)



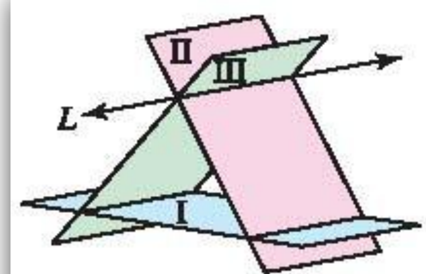
No points in common

(e)



No points in common

(g)



No points in common

(f)

$$x + y - 2z = 4$$

$$x - 3y - 4z = -2$$

$$2x + y + 2z = 0$$

$$4x + 8y + z = 2$$

$$x + 7y - 3z = -14$$

$$2x - 3y + 2z = 3$$