

A decorative graphic on the left side of the slide features a thin grey circle. A large, dark grey left square bracket is positioned to the left of the circle, and a large, dark grey right square bracket is positioned to the right of the circle. A horizontal green bar with a gradient, transitioning from a darker green on the left to a lighter green on the right, spans across the middle of the slide, partially overlapping the circle and the brackets.

5.5 Dividing Polynomials

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Objectives:

- Divide a polynomial by a monomial.
- Divide a polynomial by a polynomial of two or more terms.
- Divide polynomial functions.



Parts of a Division Problem

- There are three parts to a division problem: the **dividend**, the **divisor**, and the **quotient**.
- A division problem can be written three different ways:

$$\frac{\textit{dividend}}{\textit{divisor}} = \textit{quotient}$$

$$(\textit{dividend}) \div (\textit{divisor}) = \textit{quotient}$$

$$\textit{divisor} \overline{) \textit{dividend}}^{\textit{quotient}}$$

Dividing a Polynomial by a Monomial

To divide a polynomial by a monomial, _____
_____,
and then write each quotient in _____.

- Divide.

$$(12m^6 + 18m^5 + 30m^4) \div 6m^2$$

[Example 1]

- Divide.

$$\frac{50m^4 - 30m^3 + 20m}{10m^3}$$

[Example 2]

- Divide.

$$(-8p^4 - 6p^3 - 12p^5) \div (-3p^3)$$

[Example 3]

- Divide.

$$\frac{45x^4y^3 + 30x^2y^2 - 60x^2y}{15x^2y}$$

[Long Division]

- Divide.

$$3,257 \div 12$$

Divide: $(5x - 8 + 4x^3 - 4x^2) \div (2x - 1)$

- Step 1: Rewrite as a long division problem. Make sure that both polynomials are written in descending order, fill in any missing terms with a zero term.
- Step 2:
 - a) Take the first term of the dividend and divide by the first term of the divisor:

Place this value above it's like term.

[Dividing a Polynomial by a Polynomial]

- Step 2:

- b) Multiply the $2x^2$ by $2x - 1$ to get $1 = 4x^3 - 2x^2$.
Place this value below the like terms.

$$\begin{array}{r} 2x^2 \\ 2x - 1 \overline{) 4x^3 - 4x^2 + 5x - 8} \end{array}$$

- c) Subtract.

[Dividing a Polynomial by a Polynomial]

- Step 3: Bring down the next term.

$$\begin{array}{r} 2x-1 \overline{) 4x^3 - 4x^2 + 5x - 8} \\ \underline{-4x^3 + 2x^2} \\ -2x^2 + 5x \end{array}$$

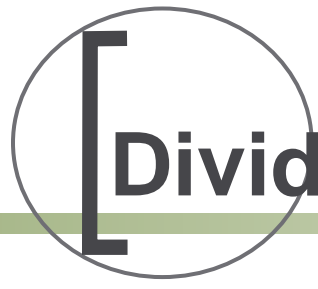
The diagram illustrates the polynomial division process. The divisor is $2x - 1$. The dividend is $4x^3 - 4x^2 + 5x - 8$. The first step shows the $4x^3$ term being divided by $2x$ to get $2x^2$, which is written above the line. This $2x^2$ is then multiplied by the divisor $2x - 1$ to get $-4x^3 + 2x^2$. This result is subtracted from the dividend, with the subtraction shown as adding the opposite: $-4x^3$ becomes $+4x^3$ and $+2x^2$ becomes $-2x^2$. The result of the subtraction is $-2x^2 + 5x$, with the -8 term brought down. The $4x^3$ term in the original dividend is crossed out with a red line.

- Step 4: Repeat steps 2 and 3 until you have brought down the last term.

[Dividing a Polynomial by a Polynomial]

- Step 5: State your answer. If there is a remainder, place the remainder over the divisor and add it to the quotient.

$$\begin{array}{r} 2x^2 - x + 2 \\ 2x - 1 \overline{) 4x^3 - 4x^2 + 5x - 8} \\ \underline{(-) 4x^3 + 2x^2} \\ -2x^2 + 5x \\ \underline{(+ 2x^2 + x)} \\ 4x - 8 \\ \underline{(- 4x + 2)} \\ -6 \end{array}$$



Dividing a Polynomial by a Polynomial

- Step 6: Check. Multiply the divisor by the quotient and add the remainder.

[Dividing a Polynomial by a Polynomial]

To divide a polynomial by a polynomial, follow the six steps outlined below.

- **Step 1:** Rewrite as a long division problem. Make sure that both polynomials are written in descending order, filling in any missing terms with a zero term.
- **Step 2:**
 - a) Divide the first term of the dividend by the first term of the divisor. Place that quotient above it's like term.
 - b) Multiply the quotient from part a by the divisor. Place that product below it's like term.
 - c) Subtract.
- **Step 3:** Bring down the next term.
- **Step 4:** Repeat steps 2 and 3 until you have brought down the last term.
- **Step 5:** State your answer. If there is a remainder, place the remainder over the divisor and add it to the quotient.
- **Step 6:** Check. Multiply the divisor by the quotient and add the remainder.



Example 4

- Divide $8x^3 - 4x^2 - 14x + 15$ by $2x + 3$.



Example 5

- Divide

$$\frac{2x^3 + 5x + x^2 + 13}{2x + 3}$$



Example 6



- Divide $4x^3 + 3x - 8$ by $x + 2$
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Example 7

- Divide

$$\frac{6m^4 + 9m^3 + 2m^2 - 8m + 7}{3m^2 - 2}$$

[Example 8]

- Divide $3x^3 + 7x^2 + 7x + 11$ by $3x + 6$

[Example 9]

- For $f(x) = 2x^2 + 17x + 30$ and $g(x) = 2x + 5$, find $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{f}{g}\right)(-1)$.