Objectives

- Define rational expressions.
- Define rational functions and describe their domains.
- Write rational expressions in lowest terms.
- Multiply rational expressions.
- Find reciprocals of rational expressions.
- Divide rational expressions.

7.1 Rational Expressions and Functions; Multiplying and Dividing

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Rational Expressions

- A rational expression is the quotient of two polynomials with denominator not equal to zero.

Rational Functions

- A function that is defined by a quotient of polynomials is called a _____ and has the form:

Example 1

For each rational function, find all numbers that are not in the domain. Then give the domain using set-builder notation.

a) $f(x) = \frac{3}{7x-14}$

b) $f(x) = \frac{x+6}{x^2-x-6}$

Example 1 (continued)

c) $f(x) = \frac{3+2x}{5}$

d) $f(x) = \frac{2}{x^2+4}$

Fundamental Property of Rational Numbers

- If $\frac{a}{b}$ is a rational number and if c is any nonzero real number, then

Writing a Rational Expression in Lowest Terms

- _____ both numerator and denominator to find their greatest common factor (GCF).
- _____. Divide out common factors.

$$\frac{4y+2}{6y+3}$$

Example 2

Write each rational expression in lowest terms.

- $\frac{y^2+2y-3}{y^2-3y+2}$
- $\frac{y^2-4}{2y+4}$

Example 3

Write each rational expression in lowest terms.

- $\frac{1+p^3}{1+p}$
- $\frac{y+2}{y^2+4}$

Example 4

Write the rational expression in lowest terms.

$$\frac{pr + qr + ps + qs}{pr + qr - ps - qs}$$

Example 5

Write each rational expression in lowest terms.

- $\frac{x-1}{1-x}$
- $\frac{p-1}{4-p^2}$

Multiplying Rational Expressions

- _____ all numerators and denominators as completely as possible.
- _____.
- _____ the numerators and multiply the denominators.
- _____ to be sure that the product is in lowest terms.

Example 6

Multiply.

$$\frac{m^2 - 16}{m + 2} \cdot \frac{1}{m + 4}$$

Example 7

Multiply.

$$(p-4)\cdot\frac{3}{5p-20}$$

Example 8

Multiply.

$$\frac{c^2 + 2c}{c^2 - 4} \cdot \frac{c^2 - 4c + 4}{c^2 - c}$$

Dividing Rational Expressions

- To divide rational expressions, *multiply* the first by the reciprocal of the second.

$$\frac{2p^2q}{3pq^4} \div \frac{pq}{6p^2q^2}$$

Example 9

Divide.

$$\frac{3k^2 + 5k - 2}{9k^2 - 1} \div \frac{4k^2 + 8k}{k^2 - 7k}$$