

8.2 RATIONAL EXPONENTS

Objectives

- Use exponential notation for n th roots.
- Define and use expressions of the form $a^{\frac{m}{n}}$.
- Convert between radicals and rational exponents.
- Use the rules for exponents with rational exponents.

EXPONENTIAL NOTATION FOR N TH ROOTS

If $\sqrt[n]{a}$ is a real number then,



EXAMPLE 1

Evaluate each exponential.

○ $32^{\frac{1}{5}}$

○ $64^{\frac{1}{2}}$

○ $-81^{\frac{1}{4}}$

○ $(-81)^{\frac{1}{4}}$



EXAMPLE 1 (CONTINUED)

Evaluate each exponential.

○ $(-64)^{\frac{1}{3}}$

○ $\left(\frac{1}{27}\right)^{\frac{1}{3}}$

○ $\left(\frac{1}{16}\right)^{\frac{1}{4}}$

○ $(-27)^{\frac{1}{3}}$



EXPONENTIAL NOTATION FOR N TH ROOTS

If m and n are positive integers with $\frac{m}{n}$ in lowest terms, then,

provided that $a^{\frac{1}{n}}$ is a real number.



FLOWER POWER



EXAMPLE 2

Evaluate each exponential.

○ $25^{\frac{3}{2}}$

○ $27^{\frac{2}{3}}$

○ $-16^{\frac{3}{2}}$

○ $(-64)^{\frac{2}{3}}$

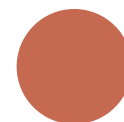


EXAMPLE 2 (CONTINUED)

Evaluate each exponential.

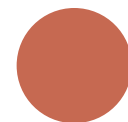
○ $(-36)^{\frac{3}{2}}$

○ $(-125)^{\frac{4}{3}}$



EXPONENTIALS WITH NEGATIVE RATIONAL EXPONENTS

If $a^{\frac{m}{n}}$ is a real number, then,



EXAMPLE 3

Evaluate each exponential.

○ $81^{-\left(\frac{3}{4}\right)}$

○ $36^{-\left(\frac{3}{2}\right)}$

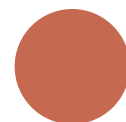


EXAMPLE 3 (CONTINUED)

Evaluate each exponential.

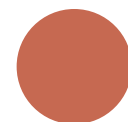
○ $\left(\frac{64}{25}\right)^{-\frac{3}{2}}$

○ $\left(\frac{216}{125}\right)^{-\frac{2}{3}}$



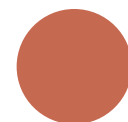
CONVERTING BETWEEN RATIONAL EXPONENTS AND RADICALS

**If all indicated roots are real numbers,
then**



CONVERTING BETWEEN RATIONAL EXPONENTS AND RADICALS

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EXAMPLE 4

Write each exponential as a radical. Assume that all variables represent positive real numbers.

○ $19^{\frac{1}{2}}$

○ $11^{\frac{3}{4}}$

○ $14x^{\frac{2}{3}}$

○ $5x^{\frac{3}{5}} - (2x)^{\frac{3}{5}}$



EXAMPLE 4 (CONTINUED)

Write each exponential as a radical. Assume that all variables represent positive real numbers.

○ $x^{-\frac{5}{7}}$

○ $(x^2 + y^2)^{\frac{1}{3}}$



EXAMPLE 5

Write each radical as an exponential.

○ $\sqrt{37}$

○ $\sqrt[3]{10}$

○ $\sqrt[4]{9^8}$

○ $\sqrt[8]{x^8}$



Rules for Rational Exponents

Let r and s be rational numbers. For all real numbers a and b for which the indicated expressions exists, the following are true.



EXAMPLE 6

Write with only positive exponents. Assume that all variables represent positive real numbers.

○ $3^{\frac{1}{2}} \cdot 3^{\frac{1}{3}}$

○ $\frac{7^{\frac{2}{3}}}{\frac{4}{7^3}}$



EXAMPLE 6 (CONTINUED)

Write with only positive exponents. Assume that all variables represent positive real numbers.

○ $\left(\frac{a^{\frac{1}{3}}b^{\frac{2}{3}}}{b}\right)^6$

○ $\left(\frac{a^3b^{-4}}{a^{-2}b^{\frac{1}{5}}}\right)^{-\frac{1}{2}}$



EXAMPLE 6 (CONTINUED)

Write with only positive exponents. Assume that all variables represent positive real numbers.

○ $r^{\frac{2}{5}}(r^{\frac{3}{5}} + r^{\frac{8}{5}})$



EXAMPLE 7

Write with only positive exponents. Assume that all variables represent positive real numbers.

○ $\sqrt[4]{x^3} \cdot \sqrt[5]{x}$

○ $\frac{\sqrt{x^5}}{\sqrt[3]{x}}$



EXAMPLE 7 (CONTINUED)

Write with only positive exponents. Assume that all variables represent positive real numbers.

○ $\sqrt[3]{6\sqrt{x}}$

○ $\sqrt{\sqrt[4]{z}}$

