8.2 Rational Exponents

Objectives

- Use exponential notation for nth roots.
- Define and use expressions of the form $a^{\frac{m}{n}}$.
- Convert between radicals and rational exponents.
- Use the rules for exponents with rational exponents.

Exponential Notation for *n*th Roots

If $\sqrt[n]{a}$ is a real number then,

Example 1

Evaluate each exponential.

- $32^{\frac{1}{5}}$
- $64^{\frac{1}{2}}$
- $-81^{\frac{1}{4}}$
- $(-81)^{\frac{1}{4}}$

Example 1 (continued)

Evaluate each exponential.

- $(-64)^{\frac{1}{3}}$
- $\left(\frac{1}{27}\right)^{\frac{1}{3}}$
- $\left(\frac{1}{16}\right)^{\frac{1}{4}}$
- $(-27)^{\frac{1}{3}}$

Exponential Notation for *n*th Roots

If m an n are positive integers with $\frac{m}{n}$ in lowest terms, then,

provided that $a^{\frac{1}{n}}$ is a real number.

Flower Power

Example 2

Evaluate each exponential.

- $25^{\frac{3}{2}}$
- $27^{\frac{2}{3}}$
- $-16^{\frac{3}{2}}$
- $(-64)^{\frac{2}{3}}$

Example 2 (continued)

Evaluate each exponential.

- $(-36)^{\frac{3}{2}}$
- $(-125)^{\frac{4}{3}}$

Exponentials with Negative Rational Exponents

If $a^{\frac{m}{n}}$ is a real number, then,

Example 3

Evaluate each exponential.

- $81^{-\left(\frac{3}{4}\right)}$
- $36^{-\left(\frac{3}{2}\right)}$

Example 3 (continued)

Evaluate each exponential.

- $\left(\frac{64}{25}\right)^{-\frac{3}{2}}$
- $\left(\frac{216}{125}\right)^{-\frac{2}{3}}$

Converting Between Rational Exponents and Radicals

If all indicated roots are real numbers, then

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Example 4

Write each exponential as a radical. Assume that all variables represent positive real numbers.

- $19^{\frac{1}{2}}$
- $11^{\frac{3}{4}}$
- $14x^{\frac{2}{3}}$
- $5x^{\frac{3}{5}} - (2x)^{\frac{3}{5}}$

Example 4 (continued)

Write each exponential as a radical. Assume that all variables represent positive real numbers.

- $x^{-\frac{5}{7}}$
- $(x^2 + y^2)^{\frac{1}{3}}$

Example 5

Write each radical as an exponential.

- $\sqrt{37}$
- $\sqrt[3]{10}$
- $\sqrt[4]{9^8}$
- $\sqrt[8]{x^8}$

Rules for Rational Exponents

Let r and s be rational numbers. For all real numbers a and b for which the indicated expressions exists, the following are true.

Example 6

Write with only positive exponents. Assume that all variables represent positive real numbers.

- $3^{\frac{1}{2}} \cdot 3^{\frac{1}{3}}$
- $\dfrac{7^{\frac{2}{3}}}{7^{\frac{4}{3}}}$

Example 6 (continued)

Write with only positive exponents. Assume that all variables represent positive real numbers.

- $\left(\frac{a^{\frac{1}{3}}b^{\frac{2}{3}}}{b}\right)^{6}$
- $\left(\frac{a^{3}b^{-4}}{a^{-2}b^{\frac{1}{5}}}\right)^{-\frac{1}{2}}$

Example 6 (continued)

Write with only positive exponents. Assume that all variables represent positive real numbers.

- $r^{\frac{2}{5}}\left(r^{\frac{3}{5}} + r^{\frac{8}{5}}\right)$

Example 7

Write with only positive exponents. Assume that all variables represent positive real numbers.

- $\sqrt[4]{x^3} \cdot \sqrt[5]{x}$
- $\dfrac{\sqrt{x^5}}{\sqrt[3]{x}}$

Example 7 (continued)

Write with only positive exponents. Assume that all variables represent positive real numbers.

- $\sqrt[3]{\sqrt[6]{x}}$
- $\sqrt{\sqrt[4]{z}}$