8.3 Simplifying Radical Expressions

Objectives

- Use the product rule for radicals.
- Use the quotient rule for radicals.
- Simplify radicals.
- Simplify products and quotients of radicals with different indexes.
- Use the Pythagorean theorem.
- Use the distance formula.

Product Rule for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and n is a natural number, then

Example 1

Multiply. Assume that all variables represent positive real numbers.

- $\sqrt{5} \cdot \sqrt{13}$
- $\sqrt{7} \cdot \sqrt{xy}$
- $\sqrt[3]{2} \cdot \sqrt[3]{7}$
- $\sqrt[6]{8r^2} \cdot \sqrt[6]{2r^3}$

Example 1 (continued)

Multiply. Assume that all variables represent positive real numbers.

- $\sqrt[5]{9y^2x} \cdot \sqrt[5]{8xy^2}$
- $\sqrt{7} \cdot \sqrt[3]{5}$

Quotient Rule for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, $b \neq 0$, and n is a natural number, then

Example 2

Simplify. Assume that all variables represent positive real numbers.

- $\sqrt{\frac{100}{81}}$
- $\sqrt{\frac{11}{25}}$
- $\sqrt[3]{\frac{18}{125}}$
- $\sqrt{\frac{y^8}{16}}$

Example 2 (continued)

Simplify. Assume that all variables represent positive real numbers.

- $-\sqrt[3]{\frac{x^2}{r^{12}}}$
- $\sqrt[5]{\frac{x}{32}}$

Conditions for a Simplified Radical

1. The radicand has no factor raised to a power greater than or equal to the index.

2. The radicand has no fractions.

3. No denominator contains a radical.

4. Exponents in the radicand and the index of the radical have greatest common factor 1.

Example 3

Simplify.

- $\sqrt{32}$
- $\sqrt{300}$

Example 3 (continued)

Simplify

- $\sqrt{35}$
- $\sqrt[3]{54}$
- $\sqrt[4]{243}$

Example 4

Simplify. Assume that all variables represent positive real numbers.

- $\sqrt{25p^7}$

- $\sqrt{72y^3x}$

Example 4 (continued)

Simplify. Assume that all variables represent positive real numbers.

- $\sqrt[3]{-27x^5y^7z^6}$
- $-\sqrt[4]{32a^5b^7}$

Example 5

Simplify. Assume that all variables represent positive real numbers.

- $\sqrt[12]{2^3}$
- $\sqrt[6]{t^2}$

Simplifying Radicals by Using Smaller Indexes

If m is an integer, n and k are natural numbers, and all indicated roots exist, then

Example 6

Simplify

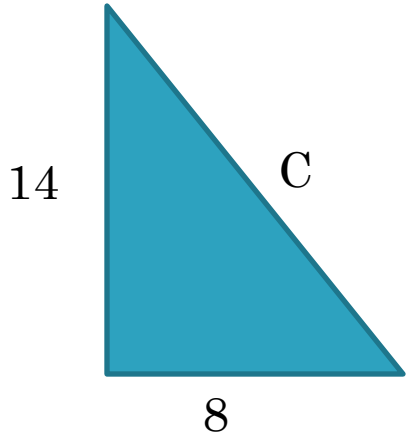
- $\sqrt{5} \cdot \sqrt[3]{4}$
- $\sqrt{7} \cdot \sqrt[4]{2}$

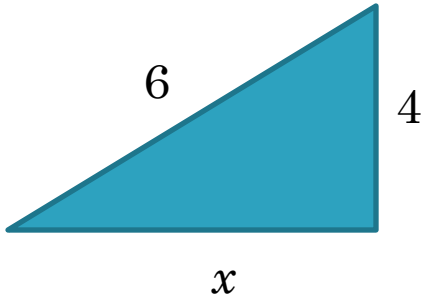
Pythagorean Theorem

If a and b are the lengths of the shorter sides of a right triangle and c is the length of the longest side, then

Example 7

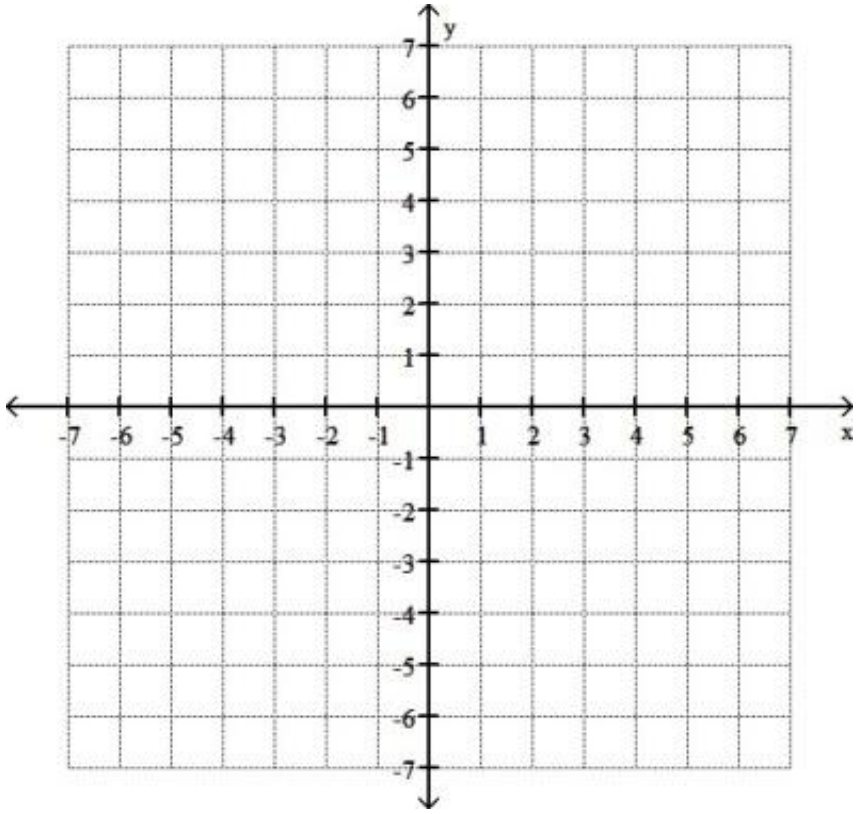
Use the Pythagorean theorem to find the length of the unknown side of the triangle.





The Distance Formula

- $(3, -4)$ and $(-5, 3)$



The Distance Formula

The distance d between the points (x_1, y_1) and (x_2, y_2) is

Example 8

Find the distance between each pair of points.

- $(2, -1)$ **and** $(5, 3)$
- $(-3, 2)$ **and** $(0, -4)$