8.7 Complex Numbers

Objectives

- Simplify numbers of the form $\sqrt{-b}$ where $b > 0$.
- Recognize the subsets of the complex numbers.
- Add and subtract complex numbers.
- Multiply complex numbers.
- Divide complex numbers.
- Find powers of i.

THE IMAGINARY UNIT i.

The imaginary unit i is defined as

That is, i is the principal root of -1.

For any positive real number b,


A
Christmas
Story

Example 1

Write each number as a product of a real number and i.

- $\sqrt{-25}$
- $-\sqrt{-81}$
- $\sqrt{-7}$
- $\sqrt{-44}$

Example 2

Multiply.

- $\sqrt{-16} \cdot \sqrt{-25}$
- $\sqrt{-6} \cdot \sqrt{-5}$
- $\sqrt{-8} \cdot \sqrt{-6}$
- $\sqrt{-5} \cdot \sqrt{7}$

Example 3

Divide.

- $\dfrac{\sqrt{-80}}{\sqrt{-5}}$
- $\dfrac{\sqrt{-40}}{\sqrt{10}}$

The Imaginary Number

- The imaginary number takes mathematics to another dimension. It was discovered in sixteenth century Italy at a time when being a mathematician was akin to being a modern day rock star, when there was 'nuff respect' to be had from solving a particularly 'wicked' equation. And the wicked equation of the day went like this: "If the square root of +1 is both +1 and –1 , then what is the square root of –1?"

- Previously, mathematicians had rolled their eyes skyward and prayed for divine intervention. But where others failed, the creative Italian Rafaello Bombelli triumphed with his invention of the imaginary number. The imaginary number is the square root of –1 and is known as *i*.

- Imaginary numbers are real numbers multiplied by i. If, like many, you find yourself saying "but what's the point?" then think on this. Imagine a world without electric circuits. No circuits, so no computers. No computers, so you wouldn't be reading this now. And while engineers need the imaginary number to analyze electrical waves, physicists need it to calculate the fundamental forces that govern our Universe via quantum mechanics.

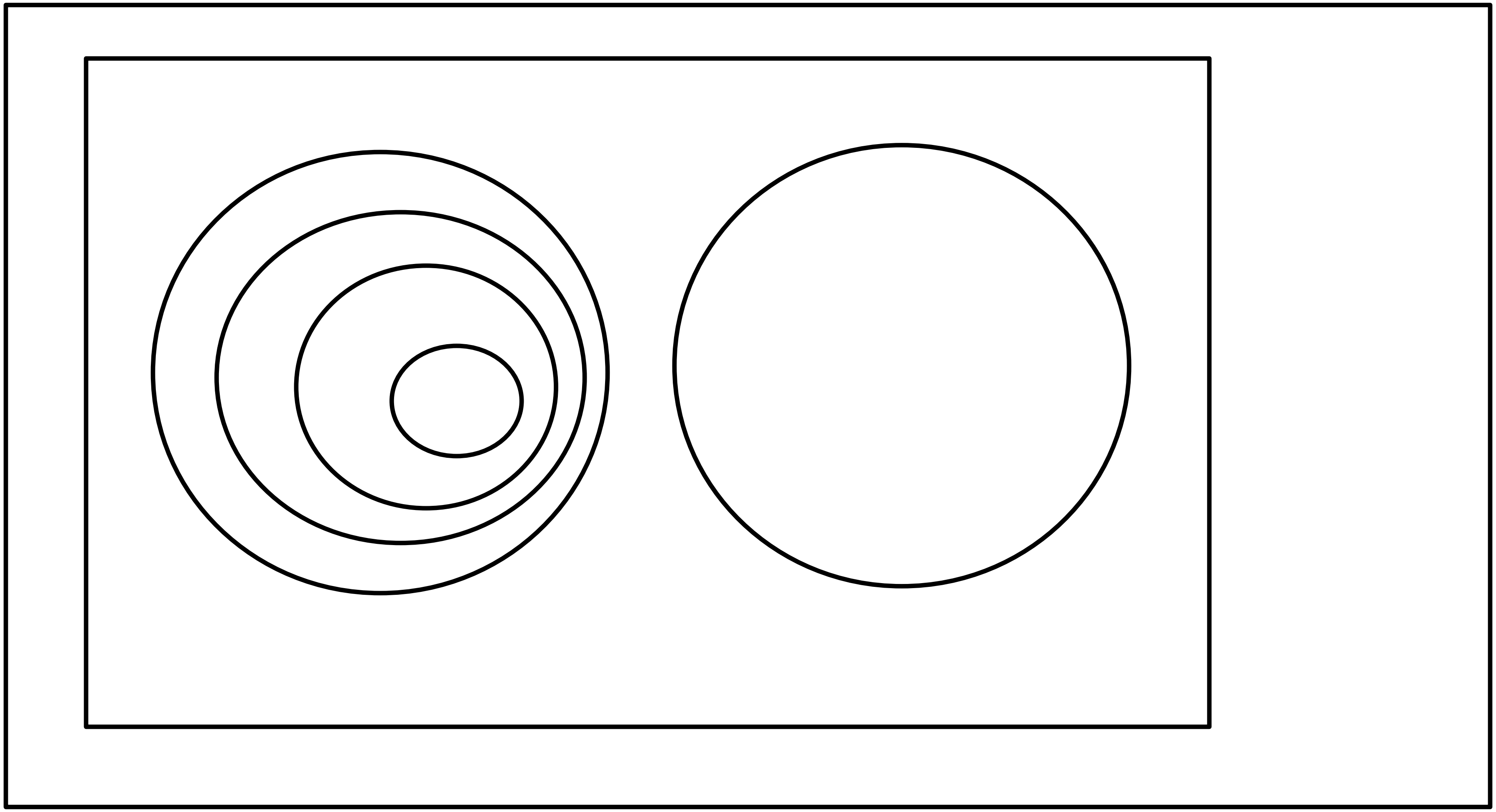
- For most human tasks, real numbers offer an adequate description of data. Fractions such as $^2/_3$ and $^1/_8$ are meaningless to a person counting stones, but essential to a person comparing the sizes of different collections of stones. Negative numbers such as -3 and -5 are meaningless when measuring the mass of an object, but essential when keeping track of monetary debits and credits. Similarly, imaginary numbers have essential concrete applications in a variety of sciences and related areas such as signal processing, control theory, electromagnetism, quantum mechanics, cartography, vibration analysis, and many others.

Complex Numbers

If a and b are real numbers, then any number of the form

is called a ________________. In the complex number $a + bi$, the number a is called the __________ and b is called the ___________ ________.

The Number System



Example 4

Add.

- $(-1 - 8i) + (9 - 3i)$

- $(-3 + 2i) + (1 - 3i) + (-7 - 5i)$

Example 5

Subtract.

- $(-1 + 2i) - (4 + i)$
- $(8 - 5i) - (12 - 3i)$
- $(-10 + 6i) - (-10 + 10i)$

Example 6

Multiply.

- $6i(4 + 3i)$
- $(6 - 4i)(2 - 4i)$
- $(3 - 2i)(3 + 4i)$

Example 7

Find each quotient.

- $\dfrac{5-i}{i}$
- $\dfrac{23-i}{3-i}$

Example 8

Find each quotient.

- $\frac{-1+5i}{3+2i}$

POWERS OF i

Powers of i

Example 9

Find each power of i.

- i^{28}
- i^{19}
- i^{-9}
- i^{-22}